

Axel Schulze-Halberg  
Department of Mathematics and Actuarial Science  
Associate Professor  
At IU Northwest since 2009

## SCHOLARLY PUBLICATIONS AND CREATIVE ACHIEVEMENTS

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### ARTICLES (in refereed journals):

- (with P. Roy) Quantum models with energy-dependent potentials solvable in terms of exceptional orthogonal polynomials, *Ann. Phys.* (2017), to appear
- (with A. Contreras-Astorga) Recursive representation of Wronskians in confluent supersymmetric quantum mechanics, *J. Phys. A* (2017), to appear
- (with B. Roy) Generalized quantum nonlinear oscillators: exact solutions and rational extensions, *J. Math. Phys.* 57 (2016), 102103
- Trigonometric potentials arising from the spheroidal equation: supersymmetric partners and integral formulas, *Eur. Phys. J. Plus* 131 (2016), 202
- Arbitrary-order Jordan chains associated with quantum-mechanical Hamiltonians: representations and integral formulas, *J. Math. Phys.* 57 (2016), 023521
- Supersymmetric partners and confinement of a spiked inverted oscillator model, *Eur. Phys. J. Plus* 130 (2015), 228
- (with J. Morris) Light wave propagation through a dilaton-Maxwell domain wall, *Phys. Rev. D* 92 (2015), 085026
- (with J. Wang) Two-parameter double-oscillator model of Mathews-Lakshmanan type: series solutions and supersymmetric partners, *J. Math. Phys.* 56 (2015), 072106
- Closed-form solutions and supersymmetric partners of the inverted Mathews-Lakshmanan oscillator, *Eur. Phys. J. Plus* 130 (2015), 134 (10pp)
- (with A. Contreras-Astorga) On integral and differential representations of Jordan chains and the confluent supersymmetry algorithm, *J. Phys. A* 48 (2015), 315202
- (with J. Wang) Confluent supersymmetric partners of quantum systems emerging from the spheroidal equation, *Symmetry* 7 (2015), 412-426
- (with D. Dec) Index dependence of Bessel integral moments, *Integr. Transf. Spec. F.* 26 (2015), 460-469
- A. Contreras-Astorga and A. Schulze-Halberg, "The generalized zero-mode supersymmetry scheme and the confluent algorithm", *Ann. Phys.* 354 (2015), 354-364

- Burgers equation with time-dependent coefficients and nonlinear forcing term: linearization and exact solvability, *Commun. Nonlinear Sci. Numer. Simul.* 22 (2015), 1068-1083
- (with B. Roy) Time dependent potentials associated with exceptional orthogonal polynomials, *J. Math. Phys.* 55 (2014), 123506
- (with A. Contreras-Astorga) The confluent supersymmetry algorithm for Dirac equations with pseudoscalar potentials, *J. Math. Phys.* 55 (2014), 103506
- (with J. Wang) Bound state solutions of the Klein-Gordon equation for the Mathews-Lakshmanan oscillator, *Few-Body Syst.* 55 (2014), 1223-1232
- (with B. Roy) Darboux partners of pseudoscalar Dirac potentials associated with exceptional orthogonal polynomials, *Ann. Phys.* 349 (2014), 159–170
- (with J. Garcia-Ravelo, A Menendez and J. Garcia-Martinez) Effective Kratzer and Coulomb potentials as limit cases of a multiparameter exponential-type potential, *Phys. Lett. A* 378 (2014), 2038–2042
- (with B. Roy) Rational extension and Jacobi-type  $X_m$  solutions of a quantum nonlinear oscillator, *J. Math. Phys.* 54 (2013), 122104 (12pp)
- (with J. Morris) An exactly-solvable three-dimensional nonlinear quantum oscillator, *J. Math. Phys.* 54 (2013), 112107 (17pp)
- Wronskian representation for confluent supersymmetric transformation chains of arbitrary order, *Eur. Phys. J. Plus* 128 (2013), 68 (17pp)
- (with J. Morris) Higher-dimensional realization of a nonlinear, one-parameter quantum oscillator, *Eur. Phys. J. Plus* 128 (2013), 54 (11pp)
- Darboux operators for linear first-order multi-component equations in arbitrary dimensions, *Cent. Eur. J. Phys.* 11 (2013), 457-469
- (with C. Gordon) Spectral properties of a confined nonlinear quantum oscillator in one and three dimensions, to appear in *J. Math. Phys.* (2013), DOI: 10.1063/1.4801812
- Comment on "Nonsingular parametric oscillators Darboux-related to the classical harmonic oscillator" by H.C. Rosu et al., *Europhys. Lett.* 101 (2013), 30001 (2pp)
- Darboux transformations for energy-dependent potentials and the Klein-Gordon equation, *Mathematical Physics, Analysis and Geometry* (2013), Volume 16, Issue 2, pp 179-193
- (with J. Garcia-Ravelo, C. Pacheco-Garcia and J.J. Pena Gil) A position-dependent mass model for the Thomas–Fermi potential: Exact solvability and relation to  $\delta\delta$ -doped semiconductors, *Annals of Physics* (2012)
- (with J. Garcia-Ravelo and A. Trujillo) Explicit formulas for generalized harmonic perturbations of the infinite quantum well with an application to Mathieu equations, *J. Math. Phys.* 53 (2012), 102101 (14pp)

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- Darboux transformations for (1+2)-dimensional Fokker-Planck equations with constant diffusion matrix, *J. Math. Phys.* 53 (2012), 103519 (11pp)
- (with J. Morris) Special function solutions of a spectral problem for a nonlinear quantum oscillator, *J. Phys. A* 45 (2012), 305301 (9pp)
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- (with J. Garcia-Ravelo, J. Pena and A. Queijeiro). Two-parameter ladder operators for spherical Bessel functions, *Appl. Math. Lett.* 23 (2010), 462-465
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