Chapter 11

Solving Qudratic Equations

Sec 11.1 – Solving by the Square Root Property

Recall that a quadratic equation is an equation that can be written in the form

$$ax^2 + bx + c = 0$$

for real numbers a, b, and c, with $a \neq 0$.

Sec 11.1 – Solving by the Square Root Property

Problem:
Solve

$$x^2 = 12$$

$$x^2 - 12 = 0$$

$$x^2 - \left(\sqrt{12}\right)^2 = 0$$

$$(x - \sqrt{12})(x + \sqrt{12}) = 0$$

$$x - \sqrt{12} = 0 \text{ or } x + \sqrt{12} = 0$$

$$x = \sqrt{12} \text{ or } x = -\sqrt{12}$$

$$x = 2\sqrt{3} \text{ or } x = -2\sqrt{3}$$

Sec 11.1 – Solving by the Square Root Property

<u>Square Root Property of Equations</u>

If k is a positive number and if $a^2 = k$, then

$$a = \sqrt{k} \text{ or } a = -\sqrt{k}$$

and the solution set is $\{\sqrt{k}, -\sqrt{k}\}$.

Sec 11.1 – Solving by the Square Root Property

Problem:
Solve

$$(p-4)^2 = 3$$

$$p-4=\sqrt{3} \text{ or } p-4=-\sqrt{3}$$

$$p=4+\sqrt{3} \text{ or } p=4-\sqrt{3}$$

Sec 11.1 – Solving by the Square Root Property

Problem:
Solve

$$(5m+1)^2 = 7$$

$$5m+1=\sqrt{7} \text{ or } 5\text{m}+1=-\sqrt{7}$$

$$5m=-1+\sqrt{7} \text{ or } 5\text{m}=-1-\sqrt{7}$$

$$m=\frac{-1+\sqrt{7}}{5} \text{ or m}=\frac{-1-\sqrt{7}}{5}$$

$$\text{The solution set is } \left\{\frac{-1\pm\sqrt{7}}{5}\right\}$$

Sec 11.1 – Solving by the Square Root Property

Problem:
Solve

$$(7z - 1)^2 = -1$$

Since the square root of -1 is not a real number, the solution set is Ø.

Sec 11.2 – Solving by Completing the Square

Problem: Solve

$$x^2 - 20x + 34 = 0$$

We try to put this into our pattern

$$(x + b)^2 = k$$

If we expand the squared term we get

$$x^2 + 2bx + b^2 = k$$

Sec 11.2 – Solving by Completing the Square

We start by rewriting the equation as

$$x^2 - 20x = -34$$

Comparing this with

$$x^2 + 2bx + b^2 = k$$

We must have

$$2b = -20$$

or

$$b = -10$$

Sec 11.2 – Solving by Completing the Square

So we add $(-10)^2$, or b^2, to both sides

$$x^2 - 20x + 100 = -34 + 100$$

Solving

$$(x - 10)^2 = 66$$

$$x - 10 = \sqrt{66} \text{ or } x - 10 = -\sqrt{66}$$

$$x = 10 + \sqrt{66} \text{ or } x = 10 - \sqrt{66}$$

Sec 11.2 – Solving by Completing the Square

Solving by Completing the Square

- If the coefficient of the squared term is *a*, divide both sides of the equation by *a*. This assures that the coefficient of x^2 is 1.
- If there is a constant term, subtract the constant from both sides.
- Take half of the coefficient of the first degree term, square it, and add the result to both sides.
- Apply the square root property to solve.

Sec 11.2 – Solving by Completing the Square

<u>Problem</u>: Solve

$$4x^2 - 24x + 11 = 0$$

Sec 11.2 – Solving by Completing the Square

- If the coefficient of the squared term is a, divide both sides of the equation by a. This assures that the coefficient of x^2 is 1.

$$4x^2 - 24x + 11 = 0$$

$$x^2 - 6x + \frac{11}{4} = 0$$

Sec 11.2 – Solving by Completing the Square

1. If there is a constant term, subtract the constant from both sides.

$$x^2 - 6x + \frac{11}{4} = 0$$

$$x^2 - 6x = -\frac{11}{4}$$

Sec 11.2 – Solving by Completing the Square

1. Take half of the coefficient of the first degree term, square it, and add the result to both sides.

$$x^2 - 6x = -\frac{11}{4}$$

$$x^2 - 6x + 9 = -\frac{11}{4} + 9 = -\frac{11}{4} + \frac{36}{4}$$

$$x^2 - 6x + 9 = \frac{25}{4}$$

Sec 11.2 – Solving by Completing the Square

1. Apply the square root property to solve.

$$x^2 - 6x + 9 = \frac{25}{4}$$

$$(x - 3)^2 = \frac{25}{4}$$

$$x - 3 = \frac{5}{2} \text{ or } x - 3 = -\frac{5}{2}$$

$$x = \frac{11}{2} \text{ or } x = \frac{1}{2}$$

Sec 11.2 – Solving by Completing the Square

<u>Problem</u>: Solve

$$2x^2 + 4x + 10 = 0$$

Sec 11.2 – Solving by Completing the Square

- If the coefficient of the squared term is a, divide both sides of the equation by a. This assures that the coefficient of x^2 is 1.

$$2x^2 + 4x + 10 = 0$$

$$x^2 + 2x + 5 = 0$$

Sec 11.2 – Solving by Completing the Square

1. If there is a constant term, subtract the constant from both sides.

$$x^2 + 2x + 5 = 0$$

$$x^2 + 2x = -5$$

Sec 11.2 – Solving by Completing the Square

1. Take half of the coefficient of the first degree term, square it, and add the result to both sides.

$$x^2 + 2x = -5$$

$$x^2 + 2x + 1 = -5 + 1$$

$$x^2 + 2x + 1 = -4$$

Sec 11.2 – Solving by Completing the Square

1. Apply the square root property to solve.

$$x^2 + 2x + 1 = -4$$

$$(x + 1)^2 = -4$$

The solution set is Ø.

Sec 11.2 – Solving by Completing the Square

Problem:
Solve

$$3x^2 + 7x = x^2 + x + 3$$

Sec 11.2 – Solving by Completing the Square

- If the coefficient of the squared term is a, divide both sides of the equation by a. This assures that the coefficient of x^2 is 1.

$$3x^2 + 7x = x^2 + x + 3$$

$$2x^2 + 6x - 3 = 0$$

$$x^2 + 3x - \frac{3}{2} = 0$$

Sec 11.2 – Solving by Completing the Square

1. If there is a constant term, subtract the constant from both sides.

$$x^2 + 3x - \frac{3}{2} = 0$$

$$x^2 + 3x = \frac{3}{2}$$

Sec 11.2 – Solving by Completing the Square

1. Take half of the coefficient of the first degree term, square it, and add the result to both sides.

$$x^2 + 3x = \frac{3}{2}$$

$$x^2 + 3x + \frac{9}{4} = \frac{3}{2} + \frac{9}{4}$$

$$x^2 + 3x + \frac{9}{4} = \frac{15}{4}$$

Sec 11.2 – Solving by Completing the Square

1. Apply the square root property to solve.

$$x^2 + 3x + \frac{9}{4} = \frac{15}{4}$$

$$\left(x + \frac{3}{2}\right)^2 = \frac{15}{4}$$

$$x + \frac{3}{2} = \pm \frac{\sqrt{15}}{2}$$

$$x = \frac{-3 + \sqrt{15}}{2} \text{ or } x = \frac{-3 - \sqrt{15}}{2}$$

Sec 11.3 – Solving by the Quadratic Formula

In the long run, completing the square can prove tedious. Therefore, we derive a formula to handle the general case.

Sec 11.3 – Solving by the Quadratic Formula

Problem:
Solve $$ax^2 + bx + c = 0$$

Sec 11.3 – Solving by the Quadratic Formula

- If the coefficient of the squared term is a, divide both sides of the equation by a. This assures that the coefficient of x^2 is 1.

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Sec 11.3 – Solving by the Quadratic Formula

1. If there is a constant term, subtract the constant from both sides.

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Sec 11.3 – Solving by the Quadratic Formula

1. Take half of the coefficient of the first degree term, square it, and add the result to both sides.

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2 - 4ac}{4a^2}$$

Sec 11.3 – Solving by the Quadratic Formula

1. Apply the square root property to solve.

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2 - 4ac}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

Sec 11.3 – Solving by the Quadratic Formula

1. Apply the square root property to solve.

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sec 11.3 – Solving by the Quadratic Formula

Problem:
Solve

$$2x^2 + x - 8 = 0$$

Sec 11.3 – Solving by the Quadratic Formula

Compare

$$ax^2 + bx + c = 0$$

$$2x^2 + x - 8 = 0$$

$$a = 2$$

$$b = 1$$

$$c = -8$$

Sec 11.3 – Solving by the Quadratic Formula

$a = 2$

$b = 1$

$c = -8$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1 - 4(2)(-8)}}{2(2)}$$

$$x = \frac{-1 \pm \sqrt{65}}{4}$$

Sec 11.3 – Solving by the Quadratic Formula

Problem: Solve

$$42x - 9x^2 = 49$$

Rewrite as

$$0 = 9x^2 - 42x + 49$$

Sec 11.3 – Solving by the Quadratic Formula

Compare $$ax^2 + bx + c = 0$$

$$9x^2 - 42x + 49 = 0$$

$$a = 9$$

$$b = -42$$

$$c = 49$$

Sec 11.3 – Solving by the Quadratic Formula

$$a = 9$$

$$b = -42$$

$$c = 49$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{42 \pm \sqrt{(42)^2 - 4(9)(49)}}{2(9)}$$

$$x = \frac{42 \pm \sqrt{1764 - 1764}}{2(9)}$$

$$x = \frac{7}{3}$$

Sec 11.3 – Solving by the Quadratic Formula

Problem:

A ball is thrown upward from the ground. Its distance in feet from the ground at t seconds is given by

$$s = -16t^2 + 64t$$

At what time will the ball be 32 feet from the ground?

Sec 11.3 – Solving by the Quadratic Formula

$$32 = -16t^2 + 64t$$

$$0 = -16t^2 + 64t - 32$$

$$0 = t^2 - 4t + 2$$ Divide by -16

$$t = \frac{4 \pm \sqrt{16 - 4(1)(2)}}{2(1)}$$

Sec 11.3 – Solving by the Quadratic Formula

$$t = \frac{4 \pm \sqrt{16 - 4(1)(2)}}{2(1)}$$

$$t = \frac{4 \pm \sqrt{8}}{2(1)} = \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$$

$$t \approx .59 \text{ and } t \approx 3.41$$