




# Chapter 7

# Functions



## Section 7.3 - Functions

Suppose there are 100 new births in a particular city on a given day. We are going to follow these babies from birth to death. For each age, we are going to record the number still living. This process defines a relationship between age and the number of people living at that age.

## Section 7.3 - Functions

Suppose we observe the following:


Age	# Living
0	100
10	98
20	94
30	88
40	80
50	70
60	58
70	44
80	28
90	10

## Section 7.3 - Functions

The data in the chart defines a set of ordered pairs:

$$P = \{(0,100), (10,98), (20,94), (30,88), (40,80), \\ (50,70), (60,58), (70,44), (80,28), (90,10)\}$$

P is called a **relation** , because it establishes a correlation between two quantities.



## Section 7.3 - Functions

The number of people living depends on the age in question. Therefore, the number of people living is called the **dependent variable** and the age is called the **independent variable**.

## Section 7.3 - Functions

Suppose there are five people in a room and you record their voting preference in the previous election.


Person	Vote
George	Republican
Martha	Democrat
Jim	Democrat
Carolyn	Republican
Martha	Republican

## Section 7.3 - Functions

The data in this chart also defines a set of ordered pairs:

$$V = \{(G,R), (M,D), (J,D), (C,R), (M,R)\}$$

$V$  is a relation as well.



## Section 7.3 - Functions

A **function** is a relation in which, for each value of the first component of the ordered pairs, there is exactly one value of the second component.

P is a function since there is only one age associated with a given number of people living.

V is not a function, since there is one person associated with more than party.

## Section 7.3 - Functions

A convenient way to specify a function is give a rule that tells how to determine the value of the dependent variable ( $y$ ) for a specific value of the independent variable ( $x$ ).

For our population example, the rule would be:

$$y = - .01x^2 - .1x + 100$$


## Section 7.3 - Functions

In a *relation*, the set of all values of the independent variable ( $x$ ) is called the **domain**; the set of all values of the dependent variable ( $y$ ) is the **range**.

In our voting example, the domain is:

{George, Martha, Jim, Carolyn}

When the relation is defined by an equation, the domain is assumed to be all real numbers that produce real numbers when substituted for the real variable.



## Section 7.3 - Functions

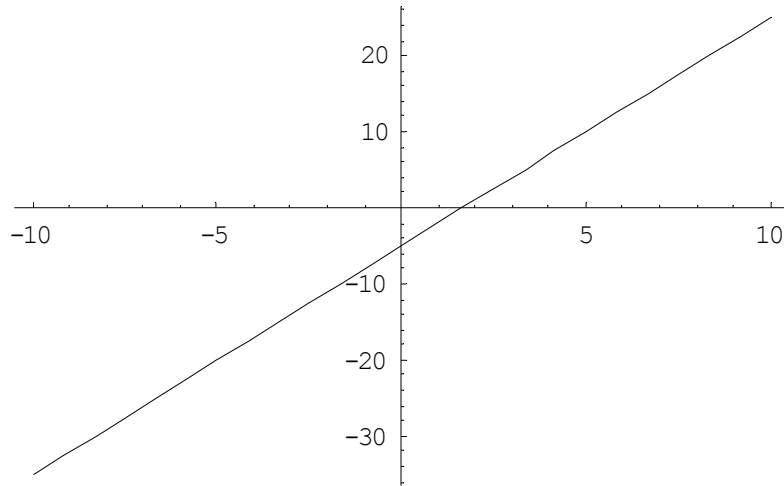
In a relation, the set of all values of the dependent variable ( $y$ ) is the **range**.

In our voting example ,the range is:

{Republican, Democrat}

# Section 7.3 - Functions

$$y = 3x - 5$$



**Domain**

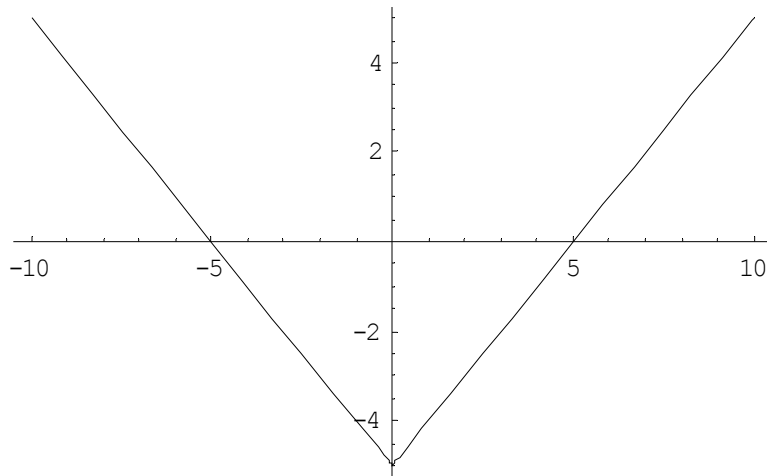
$(-\infty, \infty)$

**Range**

$(-\infty, \infty)$

# Section 7.3 - Functions

$$y = |x| - 5$$



**Domain**

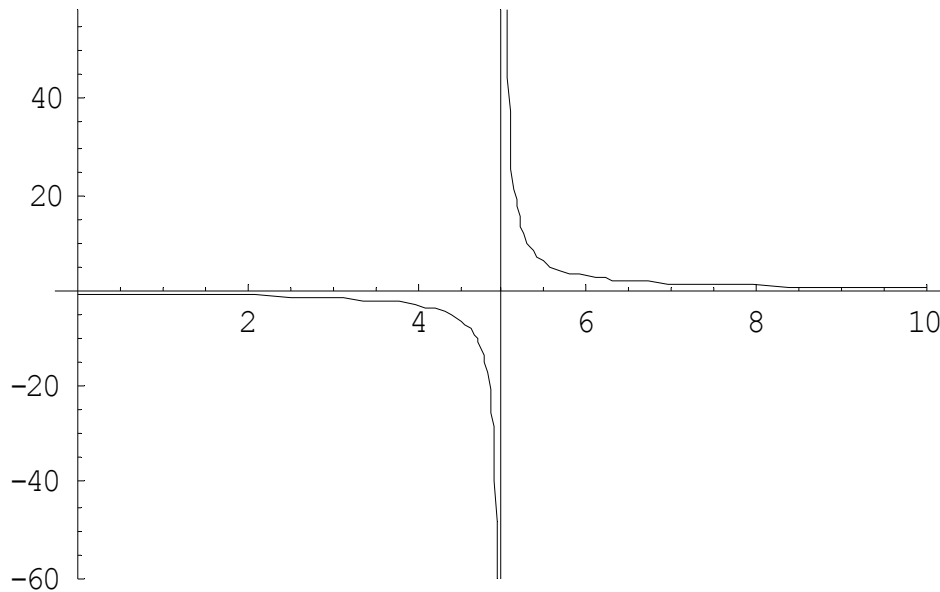
$$(-\infty, \infty)$$

**Range**

$$[-5, \infty)$$

## Section 7.3 - Functions

$$y = 3 / (x - 5)$$



**Domain**

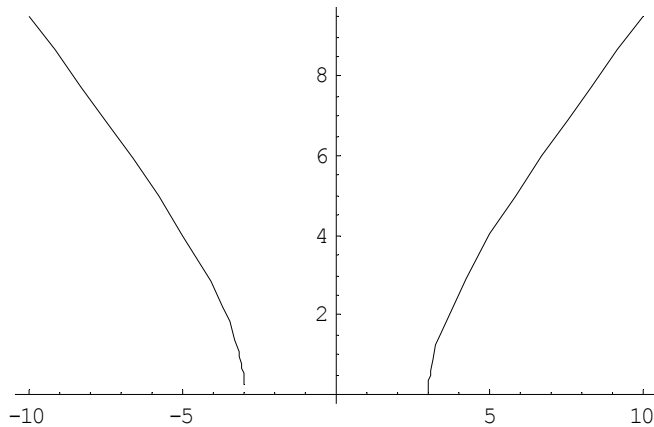
$$(-\infty, 5) \cup (5, \infty)$$

**Range**

$$(-\infty, \infty)$$

# Section 7.3 - Functions

$$y = \sqrt{x^2 - 9}$$



**Domain**

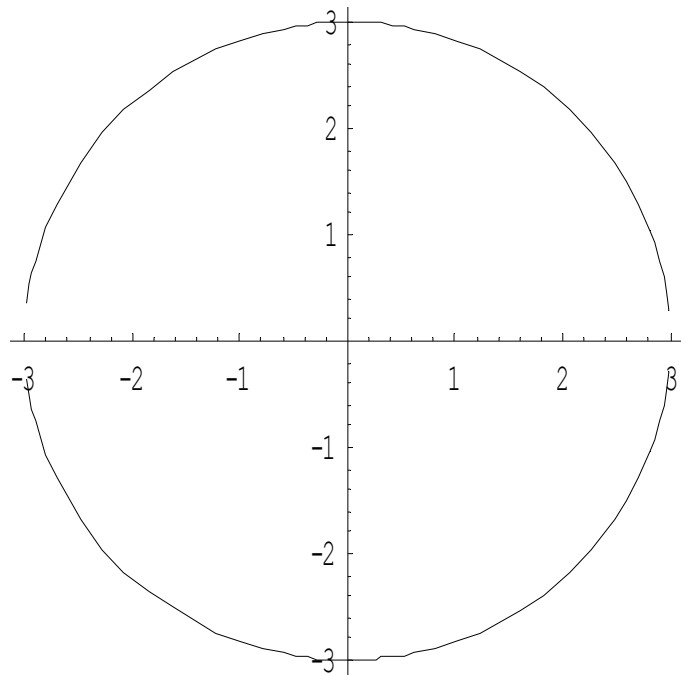
$$(-\infty, -3] \cup [3, \infty)$$

**Range**

$$[0, \infty)$$

# Section 7.3 - Functions

$$y^2 = 9 - x^2$$




**Domain**

$[-3, 3]$

**Range**

$[-3, 3]$




## Section 7.3 - Functions

In a function each value of  $x$  leads to only one value of  $y$ . Therefore, any vertical line drawn through the graph of a function intersects the graph in at most one point.

### **Vertical Line Test**

If a vertical line intersects the graph of a relation in more than one point, then the relation does not represent a function.



# Section 7.3 - Functions

## Variations of the Definition of a Function

1. A function is a relation in which, for each value of the first component of the ordered pair, there is exactly one value of the second component.
2. A function is a set of ordered pairs in which no first component is repeated.
3. A function is a rule or correspondence that assigns exactly one range value to each domain value.

## Section 7.3 - Functions

When a function is defined by a rule or an equation using  $x$  and  $y$  for the independent and dependent variables, respectively, we often write:

$$y = f(x)$$

For our population example, we would have:

$$f(x) = -.01x^2 - .1x + 100$$

## Section 7.3 - Functions

This has the advantage of explicitly showing the value of the independent variable associated with the value of the dependent variable.

For instance:

$$\begin{aligned} f(20) &= - .01(20)^2 - .1(20) + 100 \\ &= 94 \end{aligned}$$

## Section 7.3 - Functions

This helps us understand that whatever we replace the value of  $x$  by in  $f(x)$ , we insert into the value of  $x$  in the equation.

For instance:

$$f(a) = -.01(a)^2 - .1(a) + 100$$

$$f(z - 1) = -.01(z - 1)^2 - .1(z - 1) + 100$$

$$= -.01(z^2 - 2z + 1) - .1(z - 1) + 100$$

$$= -.01z^2 - .08z + 100.09$$

## Section 7.3 - Functions

We can take the replacement of  $x$  in  $f(x)$  one step further by substituting another function for  $x$ .


For instance:

$$f(x) = 5x - 1$$

$$g(x) = x^2 - 2$$

$$f(g(x)) = f(x^2 - 1) = 5(x^2 - 1) - 1$$

$$g(f(x)) = g(5x - 1) = (5x - 1)^2 - 2$$



## Section 7.3 - Functions

If an equation that defines a function is given with  $x$  and  $y$ , to find  $f(x)$ :

1. Solve the equation for  $y$ .
2. Replace  $y$  with  $f(x)$ .

## Section 7.3 - Functions

### **Problem:**

Rewrite the equation in function notation, then find  $f(1)$ .

$$x^2 - 4y = 3$$

## Section 7.3 - Functions

1. Solve the equation for  $y$ .

$$x^2 - 4y = 3$$

$$x^2 - 3 = 4y$$

$$\frac{x^2 - 3}{4} = y$$

1. Replace  $y$  with  $f(x)$ .

$$f(x) = \frac{x^2 - 3}{4}$$

## Section 7.3 - Functions

$$f(x) = \frac{x^2 - 3}{4}$$
$$f(1) = \frac{(1)^2 - 3}{4}$$
$$f(1) = -\frac{1^4}{2}$$

## Section 7.3 - Functions

A function that can be written in the form

$$f(x) = mx + b$$

for real numbers  $m$  and  $b$  is a **linear function**.

1. The domain is  $(-\infty, \infty)$ .
2. If  $m \neq 0$ , the range is  $(-\infty, \infty)$ .

## Section 7.3 - Functions

A function that can be written in the form

$$f(x) = c$$

for real numbers  $m$  and  $b$  is a **constant function**.

1. The domain is  $(-\infty, \infty)$ .
2. The range is  $\{c\}$ .