M118 – Review for the FINAL exam

1. Show work
2. You may use a calculator.
3. You may use one 8.5 x 11 sheet of paper that contains notes and formulas.
4. Last week of classes' material is in blue.

1. Write the augmented matrix for the system below and solve the system:

$$x + y + z = 2$$
$$2x + y - z = 5$$
$$x - y + z = -2$$

2. Use the Gauss- Jordan method to solve:

$$2x - 3y = 2$$
$$4x - 6y = 1$$

3. An electronics company produces three models of stereo speakers, model A, B, and C, and can deliver them by truck, van, or station wagon. A truck holds 2 boxes of model A, 2 of model B, and 3 of model C. A van holds 3 boxes of model A, 4 boxes of model B, and 2 boxes of model C. A station wagon holds 3 boxes of model A, 5 boxes of model B, and 1 box of model C. If 25 boxes of model A,33 boxes of model B, and 22 boxes of model C are to be delivered, how many vehicles of each type should be used so that all operate at full capacity?

4. Find the values of the variables in the equation:

$$\begin{bmatrix} 9 & x+y \\ r & 0 \end{bmatrix} = \begin{bmatrix} x-2y & 3 \\ 4 & 0 \end{bmatrix}$$

5. Perform the indicated operations:

$$3 \cdot \begin{bmatrix} 2 & -1 \\ 4 & 0 \end{bmatrix} - 2 \cdot \begin{bmatrix} -5 & 3 \\ -2 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} =$$

6. Find the matrix product if possible:

$$\begin{bmatrix} 5 & 2 \\ 7 & 6 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 4 & 0 \\ 2 & -1 & 2 \end{bmatrix}$$

7. Find the matrix product if possible: $\begin{bmatrix} 7 & 4 & -2 \\ -3 & 6 & 2 \end{bmatrix} \begin{bmatrix} 1 & 4 & 0 \\ 2 & -1 & 2 \end{bmatrix}$

8. Find the inverse, if it exists, for the matrix: $\begin{bmatrix} -1 & -2 \\ 3 & 5 \end{bmatrix}$

9. Solve the following systems of equations.

(a) $-x - 2y = 7$
$3x + 5y = 5$

(b) $-x - 2y = 1$
$3x + 5y = -3$

(c) $-x - 2y = 3$
$3x + 5y = -5$

(d) $-x - 2y = -1$
$3x + 5y = 1$

10. A Markov chain has the following transition matrix: $\begin{bmatrix} 0 & 0.7 & 0.3 \\ 0 & 1 & 0 \\ 0.4 & 0 & 0.6 \end{bmatrix}$. If the chain starts in state 1, what is the probability that it is in state 2 after 2 transitions?

11. Graph the following system of linear equations and list the corner points.

$$2x + y \le 10$$
$$3x \le y$$
$$x \ge 0$$
$$y \ge 2$$

12. Use graphical methods to solve the linear programming problem

Minimize $z = x + 3y$

Subject to
$x + y \le 10$
$5x + 2y \ge 20$
$-x + 2y \ge 0$
$x \ge 0, \quad y \ge 0$

13. Use graphical methods to find the maximum value of the given objective function $z = 5x + 2y$ subject to the constraints

$x + y \le 10$
$2x + y \ge 10$
$x + 2y \ge 10$
$x \ge 0, \quad y \ge 0$

14. Is $[0.3 \quad -0.1 \quad 0.8]$ a probability vector?

15. Is the following matrix a transition matrix? $\begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$

16. An insurance company classifies drivers into three groups: $G0$ (no accidents), $G1$ (one accident), and $G@$ (more than one accident). The probability that a $G0$ driver will remain a $G0$ driver after one year is 0.75 and the driver will become a $G1$ driver is 0.20. A $G1$ driver cannot become a $G0$ driver. There is a 0.70 probability that a $G1$ driver will remain a $G1$ driver. A $G2$ driver must remain a $G2$ driver.
(a) Write the transition matrix.
(b) If the insurance company only accepts new policy holders who they classify as $G0$, what will be the distribution of drivers two years from now?
(c) What is the probability that a $G0$ driver will be a $G1$ driver after three years?

17. Is the following matrix regular: $\begin{bmatrix} 0 & 1 & 0 \\ 0.4 & 0.2 & 0.4 \\ 1 & 0 & 0 \end{bmatrix}$

18. The weather in a certain spot is classified as fair, cloudy without rain, or rainy. A fair day is followed by a fair day 60% of the time, and a cloudy day 25% of the time. A cloudy day is followed by a cloudy day 35% of the time, and a rainy day 25% of the time. A rainy day is followed by a cloudy day 40% of the time, and another rainy day 25% of the time. What proportions of the days are expected to be fair, cloudy, and rainy over the long run?

19. Find the equilibrium vector for the transition matrix: $\begin{bmatrix} 0.2 & 0.8 \\ 0.3 & 0.7 \end{bmatrix}$

20. What is the augmented matrix of the system

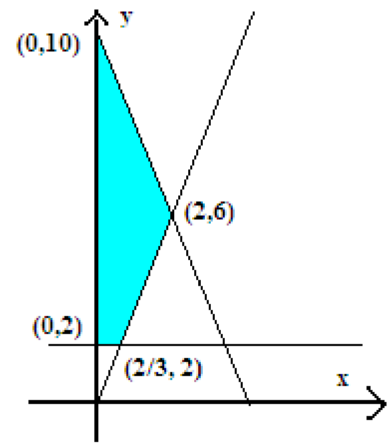
$$\begin{gathered} x + y = 3z \\ 2x + y - z = 5 \\ x - y + z = -2 \end{gathered}$$

Answers:

1. $x = 1,\ y = 2,\ z = -1$
2. The system is inconsistent and there is no solution.
3. 5 trucks, 2 vans, and 3 station wagons
4. $x = 5,\ y = -2,\ r = 4$
5. $\begin{bmatrix} 17 & -9 \\ 16 & 1 \end{bmatrix}$
6. $\begin{bmatrix} 9 & 8 & 4 \\ 19 & 22 & 12 \\ 1 & 4 & 0 \end{bmatrix}$
7. The matrix product is not possible since the number of columns in the first matrix is not equal to the number of rows in the second matrix.
8. $\begin{bmatrix} 5 & 2 \\ -3 & -1 \end{bmatrix}$

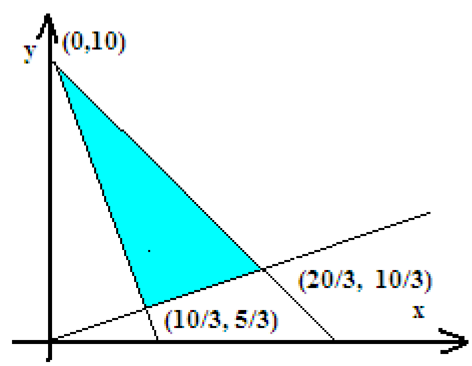
9. (a) $x = 45,\ y = -26$ (b) $x = -1,\ y = 0$ (c) $x = 5,\ y = -4$ (d) $x = -3,\ y = -2$

10. Probability is 0.7

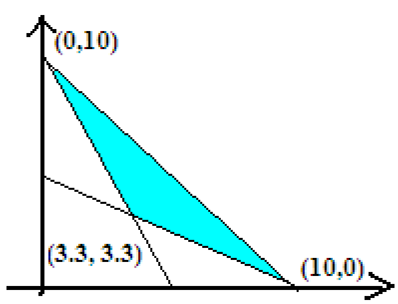


11. The corner points are (0, 2), (0, 10), (2, 6), (2/3 , 2). See picture to the left.

12. Minimum is $^{25}/_{3}$.
See picture to the right.



13. 50 at (10,0). See bellow.



14. No. Although the elements add up to 1.0, they all must lie in the interval [0,1].

15. Yes

16. Insurance problem

(a) $\begin{bmatrix} 0.75 & 0.2 & 0.05 \\ 0 & 0.7 & 0.3 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0.5625 & 0.29 & 0.1475 \end{bmatrix}$ (c) 0.3155 17. Yes

18. Fair 48.7%, cloudy 31.1%, and rainy 20.1%.

19. $\begin{bmatrix} 8/11 & 3/11 \end{bmatrix}$ 20. $\left[\begin{array}{ccc|c} 1 & 1 & -3 & 0 \\ 2 & 1 & -1 & 5 \\ 1 & -1 & 1 & -2 \end{array}\right]$