COURSE: **M403 Introduction to Modern Algebra**

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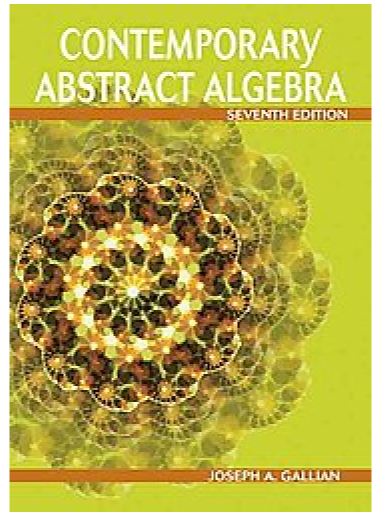

Home Page: http://www.iun.edu/~mathiho/
Time and location: **Tue – Thu 4:00 – 5:15 p.m. in HH330**
TextBook: **Gallian, Contemporary Abstract Algebra (7th Ed)**

OFFICE HOURS: When seeking instructor's help during office hours, it would be helpful if you would organize your questions in advance as much as possible, and be prepared to show your own homework attempts.

TUE	THU	FRI
By Appointment 2:15 – 4:00	By Appointment 2:15 – 4:00	By Appointment

Homework is the crucial part of the course. It is not the answer that is important but how you get there and the understanding and justification of each step. Homework problems will be collected and (some) will be graded; all problems are expected to be completed. Homework is an essential activity for you to achieve success in this class. To receive full credit for homework, it must be legible, complete, and correct. No late homework will be accepted for full credit without prior permission or valid documented excuse.

Grading:	**Homework:**	**56%**
	Midterm	**22%**
	Final Exam	**22%**

You will get full credit if you demonstrate a good attempt to solve every assigned problem. A "good attempt" means reading the problem, trying to make sense of the problem in various ways, perhaps listing what is given or trying to explain the problem in your own words. You should have written down something substantive for every problem. You will not get credit for the problem number and a question mark! Some of the assigned problems have answers in the back of the book. Therefore, you will not be graded only for correctness, and answers simply copied from the back of the book will get no credit.

Notebook

You will want to have an organized notebook because you will be allowed to use it for tests. A three-ring binder is strongly recommended since that will make it easier for you to rearrange sections in case class notes get out of order. My suggestion for organization would be that each section of the text be represented in your notebook by a collection that consists of:

a. your notes on the reading (Important!)
b. homework problems for that section
c. a list of questions you want to ask in class
d. any notes on that section taken in class.

Strategies for Taking Open Notebook Tests

1. Take good notes on the reading and in class since you are allowed to use your notebook for tests.
2. Organize your notes well enough that you can find things in a hurry. (A number of students use post-its to "flag" certain key ideas in their notebooks.)
3. Study as if the test were not open notebook; it saves time if you don't have to look up everything. Studying also helps you remember where things are if you do need to look them up. In other words, don't try to learn the material during the test.
4. When you get to the test, try to adopt an attitude as relaxed as if this were a homework assignment. Don't panic; that will just interfere with your cognitive processes. Some students tell themselves they just don't care how well they do, and they say that frees their minds to do their best work.
5. Do the easiest and shortest problems first; then go back to the other problems.
6. Write notes of explanation to the teacher on the test if:
    a. a problem seems ambiguous to you (write out what makes it ambiguous), or
    b. you wish to clarify your answer to make sure the teacher knows you understand, or
    c. you can explain how to do a similar problem but are stuck on this one.

Nature of this class:

M 403 is a math course that will introduce student to the traditional topics of abstract algebra (groups, rings, and fields). A major goal is to develop mathematical maturity by gradual introduction and development of concepts and careful and rigorous treatments of definitions and proofs. This course will prepare students for higher-level mathematics or computer science courses. To add an applied flavor we will present concepts and methodology that is used by working physicists, chemists, and computer scientists, as well as mathematicians. After finishing the course a student should be able to do computations and to write proofs. We will have abundance of exercises to develop both skills.

Prerequisite: Math M301, Math M393, or consent of instructor.

Other Class info: In case of absence check http://www.iun.edu/~mathiho/teaching.shtml for the latest homework, practice tests, test scores, class announcements…

ADVICE FOR STUDENTS FOR LEARNING ABSTRACT ALGEBRA (by Joseph A. Gallian)

Unfortunately many students struggle with this subject. First and foremost you must study the material regularly. Don't wait until a day or two before an exam or when homework is due.

One of the greatest problems I see with students is that they do not have the definitions and the theorems memorized. They will come to my office and say that they can't do a certain problem such as prove a set is a subgroup. I will say to them "What does the One Step Subgroup Test say?" They won't know. Or I will ask them "What does $|a| = 6$ mean?" They will reply $a^6 = e$. Well, of course, if you do not know the definitions and theorems you won't be able to use them.

Here is how I suggest that you learn to do proofs. Pick out several of the easier proofs that are given in the book. Write the statements down but not the proofs. Then see if you can prove them. Students often try to prove a statement without using the hypothesis. Keep in mind that you MUST use the hypothesis. If you cannot prove the statement, look at the first line of the proof in the text. See if that is enough to get you started. If it is not, then look at the next line. Repeat this over and over again until you can do them without looking at the text. Eventually you will get the hang of it. There is a direct relationship between your understanding of the subject and your ability to do proofs. Proofs test your understanding. They also test your creativity.

Keep in mind that you can only use what you have. For example, Exercise 12 in Chapter 3 says that if you have an Abelian group with two elements of order 2 then it has a subgroup of order 4. So we can let a and b be the two elements of order 2. Now all we have are a and b and the group axioms so USING ONLY a and b you must create a subgroup of order 4. Well, the axioms tell us that the identity is in the subgroup and closure tells us that ab is in there too so the subgroup must be $\{e, a, b, ab\}$. Then all we need do is to show that ab is distinct from the other three elements and use the Finite Subgroup Test to prove that this set is a subgroup. Here is another example. Look at Exercise 60 on page 69. This exercise says to prove that every finite group with more than one element must have an element of prime order. Since the group has more than one element we may let a be a nonidentity element. If a has prime order we are done. If a does not have prime order then USING only a we must find an element of prime order. We must use a to do this since a is all that we have. Since all we have to work with is a the element of prime order has to be found using a. So, consider a, a^2, a^3, a^4 etc. One of these must have prime order. But how do we know which one? Well, try some abstract examples (not specific examples such as U(n) or Dn) by saying $|a| = 15$. Then we see that $|a^3| = 5$. If we had $|a| = 12$, then $|a^4| = 3$. After trying several such examples we realize that if we write $|a| = pk$ where p is prime and $k > 1$ we have $|a^k| = p$. Note that we were able to do this problem only knowing that we had an element a and using closure.

Another thing that you must do is learn in WORDS what each concept given in symbols means. For example, Z(G) is the set of all elements that commute with EVERY element of G while C(a) is the set of all elements that commute with the particular a. Likewise, you should think of $|a|$ as the SMALLEST positive power of a that gives the identity.

Here are some remarks about how to do algebra problems.

1. Never assume a group is Abelian. Some people begin their argument for Exercise 16 of Chapter 2 by saying "Assume that the group is Abelian." This is incorrect for you have no reason to assume a group is Abelian. Many groups are not Abelian.
2. Never divide group elements. Instead, use cancellation or inverses.
3. If you use "Let" such as "Let $n-1$ belong to U(n)" then you must prove that $n-1$ is actually in U(n).

4. After you finish a proof look to see if you have used all the hypotheses. For example, if you were given that the group is Abelian check to see if you used that condition in your argument. If the group is finite check to see where you used finiteness. Occasionally, it may be the case that a given condition is not really needed but was there just to make the problem easier but usually all the given conditions are needed for you to be able to give a valid proof with what you know at this point in the book.
5. You cannot solve a problem such as Chapter 2 Exercise 12 by selecting a specific value for *n*. The problem must be done for all *n* > 2, not a few examples. In general, you cannot prove a statement is true by using an example.
6. When ask to provide an example to illustrate something, D_4 is often a good group to try. For example, Exercises 6 and 16 of Chapter 2.
7. On problems such as Exercise 20 of Chapter 2 or Exercise 14 of Chapter 4 do not just give an answer. Show that your answer is valid. You must give reasons or an explanation of why your answer is correct.
8. In many cases problems can be solved by simply writing out the expressions. For example in Exercise 26 of Chapter 2 write out $(ab)^2 = a^2 b^2$ as $abab = aabb$. Exercise 19 in Chapter 2 works the same way. Just write the expression out.
9. When you are asked to prove a statement you must not assume that the statement is true.
10. Whenever you say "Assume ..." you must have a reason why you may assume what it is you are assuming. For example, if you are given that H is a subgroup of G you may make the statement: Assume x is an element of H because subgroups are not empty. You can not say "Assume G is Abelian" without providing some reason why you may assume that G is Abelian. As another example, if you are given that a group is finite and a is an element of the group you may say "Assume $|a| = n$" because all elements of a finite group have finite order. However, if you do not know that the group is finite you can't assume that an arbitrary element from the group has finite order. Instead, you should take two cases. Case 1: $|a|$ is finite and Case 2: $|a|$ is infinite.
11. When asked for an example of something, use a specific example. For instance, in response to Exercise 6 of Chapter 2 some people say that matrices have the property that $a^{-1}ba$ is not equal to b. But you must actually give the matrices since some matrices have the desired property and some do not have the property.
12. In general, you cannot take roots (square roots, cube roots, etc.) in groups. Only integer powers of group elements are permissible.
13. When doing a problem about the order of an element, for example proving that an element and its inverse have the same order, you will usually have to deal with the finite case and infinite case separately. That is, $|a| = n$ is one argument and $|a|$ is infinity is a different case. This is usually true as well when dealing with the order of a group. The cases of a finite group and an infinite group may require different arguments.
14. When an exercise says prove something is true for an integer do not assume the integer is positive. In general, the cases that an integer is positive and an integer is negative require slightly different arguments. Usually, you can use the positive integer case to prove the negative integer case by using the Law of Exponents. To illustrate the technique consider Exercise 19 in Chapter 2. To prove $(a^{-1}ba)^n = a^{-1}b^n a$ for all *n* first prove it for positive *n* by writing out the expression $a^{-1}ba$ *n* times and cancelling all the inner *a* and a^{-1} terms. (Alternatively, you could use induction.) Now to prove the statement when *n* is negative observe that $a^{-1}b^n a = ((a^{-1}ba)^{-n})^{-1}$ and that $-n$ is positive. So, since you have already done the case when the exponent is positive you have $a^{-1}b^n a = ((a^{-1}ba)^{-n})^{-1} = (a^{-1}b^{-n}a)^{-1}$. Then using the socks-shoes property you have $(a^{-1}b^{-n}a)^{-1} = a^{-1} b^n a$. Finally, the case that $n = 0$ follows because any element to the 0th power is the identity by definition.
15. When dealing with an abstract group (that is, one in which the elements and operation are not specified) use *e* to denote the identity and use multiplication as the operation.
16. If you argue by contradiction, don't end it by saying "a contradiction." You must indicate what you are contradicting (usually this will be the hypothesis or a theorem).
17. The negation "for all" is "there exist some." For example, in an Abelian group $ab = ba$ for all *a* and *b*. So, in a non-Abelian group there exist some *a* and *b* such that *ab* is not *ba*.
18. In the text it is usually the case that elements of a group are denoted by letters from the beginning of the alphabet *a, b, c* or end of the alphabet *x,y,z*. Integers such as exponents and orders of elements or groups are usually denoted with letters from the middle of the alphabet *i,j,k,m,n,s,t*. For example, let $|a| = n$. You should use the same conventions.
19. NEVER use "if and only if" arguments when the statement is not an "if and only if" statement. Your argument is likely to be wrong since most statements are not "if and only if" and even when they are most of the time "if and only if" arguments are more difficult to make.
20. When you are given an "if and only if" statement to prove it is highly recommended that you do not use an "if and only if" argument. They are tricky to get correct for beginners. Instead, if you are asked to prove the A is true if and only if B is true. Assume that A is true and use this assumption to prove B is true. Then begin all over by assuming that B is true and use that to prove A is true. So, in the end you will have two independent proofs.

21. Please keep in mind that if you are given condition A and asked to prove condition B, you will start your proof with condition A and the last line of your proof will be condition B. If you use a proof by contradiction you can assume that A is true and that B is false to lead to a contradiction.
22. When asked to find the inverse of an element, always check your answer by multiplying the element and its purported inverse to see if you get the identity.
23. Many students make the mistake of assuming that a group is finite when that condition was not stated.
24. When you are asked to prove an "or" statement such as "Prove condition A or condition B" you begin by assuming one of them is false (you can pick either condition) and use that to prove the other condition is true. (If you assumed that condition A is false and proved condition B is true there is no need to then assume that condition B is false and prove condition A is true.) An example of this is given for Part 4 of the Lemma in Chapter 7. Exercise 11 of Chapter 7 is another example. Here we may assume that H is not R^+ (if H = R^+ we are done) and use this assumption to prove that H = R*. Another way to prove an "or" statement is to assume both conditions are false and obtain a contradiction.
25. Whenever you are asked to prove a set A is equal to a set B, proceed by assuming some element *x* belongs to A and show that *x* belongs to B. Then assume some element x belongs to B and prove that x belongs to A. For example, Exercise 20 of Chapter 3 says "Prove $C(a) = C(a^{-1})$." So, begin by assuming that *x* belongs to $C(a)$ and use this assumption to prove that *x* belongs to $C(a^{-1})$. Then assume that x belongs to $C(a^{-1})$ and use this assumption to prove that x belongs to $C(a)$
26. Proving a mapping is "onto" causes confusion among many students. If you wish to prove that some function f from A to B is onto, let *b* denote any element of B. You must find some *x* in A such that $f(x) = b$ (think of *b* as given and *x* as an unknown). To do this replace $f(x)$ by the actual formula for $f(x)$ and then solve for *x* in terms of *b*. You must check to see whether the solution you obtained is in set A. Here is an example. Say you are asked to prove that $f(x) = x^2$ from the positive reals to the positive reals is onto. We let *b* be any positive real. Then we must solve the equation $x^2 = b$ for *x*. Note that $x = \sqrt{b}$ and *x* is a positive real so we have proved that *f* is onto. In contrast, if we have the same function from the positive rationals to the positive rationals the function is not onto since the square root of a positive rational need not be a positive rational.
27. When ask to prove two groups are not isomorphic students often show that some specific mapping does not satisfy the definition of isomorphism. This proves nothing. Instead, one must show that NO mapping satisfies the definition. This can be done by assuming there is some generic isomorphism and using only properties of isomorphisms derive a contradiction. Examples 5 and 6 of Chapter 6 illustrate how this can be done. Notice that no specific mapping was assumed. Usually the easiest way to prove that two groups are not isomorphic is to show that they do not share some group property. For example, the group of nonzero complex numbers under multiplication has an element of order 4 (the square root of -1) but the group of nonzero real numbers do not have an element of order 4. As another example, we see that S4 is not isomorphic to D_{12} because D_{12} has an element of order 12 whereas S_4 has elements of orders only 1, 2, 3 and 4. Often it is easiest to proceed by checking if the largest order of any element in each of the groups agree. When the orders of the elements in two groups match you can prove they are not isomorphic by showing that they have a different number of elements of some specific order. Exercise 35 of the Supplemental Exercises for Chapters 5-8 is such a case. When comparing the number of elements of some specific order, elements of order 2 is often a good choice.

ADVICE FOR STUDENTS FOR LEARNING PROOFS

You should periodically reread this essay as the course progresses since many of the comments refer to situations that will arise from time to time. Keep it on hand when you do home work.

Proofs are constructed by utilizing definitions, theorems and facts. So, to be able to do proofs you must have the relevant definitions, theorems and facts memorized. When a new topic is first introduced proofs typically use only definitions and basic math ideas such as properties of numbers. Once you have learned some theorems about a topic you can use them to proofs more theorems.

To learn how to do proofs pick out several statements with easy proofs that are given in the textbook. Write down the statements but not the proofs. Then see if you can prove them. Students often try to prove a statement without using the entire hypothesis. Keep in mind that you MUST use the hypothesis. If you cannot prove the statement, look at the first line of the proof in the text. That might be enough to get you started. If it is not, then look at the next line and so on. Practice proving the statements you selected until you can do the proofs without looking at the text. One you have mastered your original selections pick a few new ones and practice those. There is a direct relationship between your understanding of the subject and your ability to do proofs. Proofs test your understanding. They also test your creativity.

HOW TO GET STARTED

Begin a proof by rewriting what you are given and what you are asked to prove in a more convenient form. Often this involves converting word to symbols and utilizing the definitions of the terms used in the statements. An example is "Prove that the product of two nonzero real numbers is nonzero." This converts to "If a and b are nonzero real numbers, prove that $ab \neq 0$." Begin the proof with "Assume that $a \neq 0$ and $b \neq 0$. Prove that $ab \neq 0$." (We provide a proof of this statement in the section on proof by contradiction.) It is important to begin by rewriting both the assumptions and the conclusions since this emphasizes that the former is what you have to work with and the latter is your goal.

Examples of converting words to symbols are:
n is an even integer converts to $n = 2t$ for some t
n is an odd integer converts to $n = 2t + 1$ for some t
n is a rational number converts to $n = a/b$ where a and b are integers
n is a divisor of m converts to $m = nt$ for some integer t
n is a square converts to $n = t^2$ for some integer t.

DIRECT PROOF

In a direct proof you are given one or more conditions and are asked to prove some conclusion. For proofs in abstract algebra you are permitted to use the given conditions as well as axioms, definitions and standard facts about real numbers, complex numbers, high school algebra, and linear algebra without elaboration. In a direct proof of a statement of the form A implies B, you start your proof by assuming that A is true and go through a series of steps ending with B.

As an example, consider the statement "The sum of two rational numbers is rational." To prove this we use the definition of a rational number and convert the words to expressions by recasting the statement as "If a, b, c and d are integers and $b \neq 0$ and $d \neq 0$ are not 0, then $a/b + c/d$ has the form m/n where m and n are integers." To prove this statement we observe that since $a/b + c/d = (ad + bc)/bd$ and $ad + bc$ is an integer and $bd \neq 0$, the proof is complete.

PROOF BY CONTRADICTION

Proof by contradiction is a natural way to proceed when negating the conclusion gives you something concrete to manipulate. To prove the statement "A implies B" by contradiction, begin by assuming that A is true and B is not true and end by arriving at some contradiction (possibly contradicting statement A). For example, a statement such as "Prove that $\log_2 3$ is irrational" is an obvious choice for proof by contradiction since assuming that $\log_2 3$ is rational allows you to write $\log_2 3 = m/n$ where m and n are integers. From this we have $3 = 2^{m/n}$ and therefore $3^n = 2^m$. Since the right side is even and the left side is odd we have contradicted a basic fact about integers. If you argue by contradiction, don't end it by saying "a contradiction." You must indicate what you are contradicting (usually this will be the hypothesis, a theorem or a fact).

Here is an example where we contradict the original assumption. To prove the statement "The sum of a rational number and an irrational number is irrational" by contradiction, we let a be a rational number and b an irrational number and assume that $a + b$ is rational. But then $(a + b) + (-a) = b$ is rational. This is a contradiction to the assumption that b is irrational.

Over 2000 years ago Euclid proved that are infinitely many primes by assuming that there are only finitely many. By doing so he was able to take their product to arrive at a contradiction.

PROVING AN "OR" STATEMENT

When you are asked to prove an "or" statement such as "... prove statement A or statement B" you begin by assuming one of A or B is false and use that to prove the other statement is true. It does not matter which of the statements A or B you assume to be false. If you assume A is false and are not able to prove B is true, then assume B is false and try to prove that A is true. Proving one of these two possibilities is a complete proof. There is no need to do both.

Another way to prove an "A or B" statement is to assume both statement A and statement B are false and obtain a contradiction. The statement "If a and b are nonzero real numbers, prove that ab is nonzero" is a perfect candidate for proof by contradiction since the assumption that ab = 0 allows you to take advantage of a special property of 0. To prove $ab \neq 0$ we assume that $a \neq 0$, $b \neq 0$ and $ab = 0$. Since $b \neq 0$, we know b^{-1} exists. Then $a = a(bb^{-1}) = (ab)b^{-1} = 0$, which contradicts the assumption that $a \neq 0$.

Here is another example. "If m and n are integers and mn is even, prove that m or n is even." To prove this we assume that mn is even and m and n are odd. Then we may write m = 2s + 1 and n = 2t + 1 for some integers s and t. Then mn = (2s + 1)(2t + 1)= 4st + 2s + 2t + 1 = 2(2st + s + t) + 1, which is odd. Since this contradicts the assumption that mn is even, the proof is complete.

PROOF BY CASE ANALYSIS

A common way to construct a direct proof is to examine all possible cases. Consider the statement "If the product of two integers is odd, then both of them are odd." We begin by converting words to symbols by denoting the two integers by m and n and consider four cases

CASE 1. m and n are even. In this case we can write m = 2s and n = 2t for some s and t. Then mn = 2s2t = 2(2st) and mn is even.

CASE 2. m and n are odd. In this case we can write m = 2s + 1 and n = 2t +1 for some s and t. Then mn = (2s + 1)(2t + 1) = 4st + 2s + 2t + 1 = 2(2st + t + s) + 1 and mn is odd.

CASE 3. m is even and n is odd. In this case we can write m = 2s and n = 2t + 1 for some s and t. Then mn = 2s(2t + 1) = 4st + 2s = 2(2st + s) and mn is even.

CASE 4. m is odd and n is odd. This case is the same as Case 3 since m and n are interchangeable.

To complete the proof we observe that the only case that does not yield a even product is when both m and n are odd.

PROOF BY EXPERIMENT

Although you cannot generally prove statements by experiment, many proofs can be done with the help of experimenting. One typically looks at simple cases to gain insight and this insight results in a proof.

Consider the statement "Every odd integer is the sum of two consecutive integers." Trying a few small cases we have

3 = 1 + 2

5 = 2 + 3

7 = 3 + 4.

It seems that a general pattern is 2n + 1 = n + (n+1) and indeed this gives us a proof.

Here is another example. Consider the statement "Prove that every positive odd integer is the difference of two squares." Since the statement of the problem tells us that we must look at differences of two squares, we begin by listing the small squares and taking some differences to see if we can detect a pattern. The first six squares are:

$0^2 = 0$

$1^2 = 1$

$2^2 = 4$

$3^2 = 9$

$4^2 = 16$

$5^2 = 25$.

Taking differences of successive squares we have:

$1^2 - 0^2 = 1$

$2^2 - 1^2 = 3$

$3^2 - 2^2 = 5$

$4^2 - 3^2 = 7$

$5^2 - 4^2 = 9$.

Although that it appears that by taking the difference of successive squares we will obtain every odd positive integer we still must prove that this is the case. Observing that $(n + 1)^2 - n^2 = n^2 + 2n + 1 - n^2 = 2n + 1$ is the entire proof. Moreover, this proof is valid for all odd integers, not just the positive odds.

IF AND ONLY IF PROOFS

When trying to prove an "if and only if" statement it is highly recommended not use an "if and only if" argument. They are tricky to get correct for beginners. Instead, if you are asked to prove that A is true if and only if B is true, first assume that A is true and use this assumption to prove B is true. Then begin all over by assuming that B is true and use that to prove A is true. This approach requires two independent proofs.

PROVING TWO SETS ARE EQUAL

Whenever you are asked to prove a set A is equal to a set B, proceed by assuming an element x belongs to A and use the defining property of A to show that x belongs to B. Then assume some element x belongs to B and use the defining property of B to prove that x belongs to A.

Here is an example. To prove that $\{(n+1)^2 - n^2 \mid$ where n is an integer$\}$ is the set of all odd integers we let $(n + 1)^2 - n^2$ be any member of the left side. Since $(n + 1)^2 - n^2 = n^2 + 2n + 1 - n^2 = 2n + 1$ we have shown that $(n + 1)^2 - n^2$ is a member of the right side. Now let k be any member of the right side. Since k is odd it can be written in the form 2n + 1 for some integer n and since $2n + 1 = (n+1)^2 - n^2$ we have shown that k is a member of the left side.

DISPROVING

Although "proof by example" is not legitimate, you can disprove statements by way of a single example. Consider the statement "The sum of two irrational numbers is irrational." To disprove this statement we simply observe that $\sqrt{2}$ and $-\sqrt{2}$ are irrational but $\sqrt{2} + -\sqrt{2} = 0$ is rational.

PROVING UNIQUENESS

To prove an object is unique assume that a and b are two objects with the desired property and show this property together with other known information to show that a = b. To illustrate, consider the statement "For any real number r the equation $x^3 = r$ has a unique real number solution." To prove this statement assume that a and b are both solutions of $x^3 = r$ and use algebra and properties of real numbers to prove that a = b.

LOOK BACK

After you complete a proof, look back to see if you used all the hypotheses. Also, be sure that you have provided reasons for each step.

NEGATING STATEMENTS

Be careful with negations. The negation of "for all" is "there is at least one" and vice versa. For example, the negation of the statement "For every real number x, $x^2 > 0$ " is "There exist at least one real number x for which $x^2 \leq 0$." Conversely, the negation of "There exist at least one real number x for which $x^2 \leq 0$" is "For every real number x, $x^2 > 0$." These are easy to remember by thinking of a statement such as "Everyone passed the exam." The negation is "At least one person failed the exam." The negation of "At least one person failed the exam" is "Everyone passed the exam."

PROVING A FUNCTION IS ONTO

Proving a function is "onto" causes confusion among many students. If you wish to prove that some function f from A to B is onto, let b denote any element of B. You must find some x in A such that f(x) = b (think of b as given and x as an unknown). To do this replace f(x) by the actual formula for f(x) and then solve for x in terms of b. You must check to see whether the solution you obtained is in set A. Here is an example. Say you are asked to prove that $f(x) = x^2$ from the positive reals to the positive reals is onto. We let b be any positive real. Then we must solve the equation $x^2 = b$ for x. Noting that $x = \sqrt{b}$ is a positive real solution proves that f is onto. In contrast, if we have the same function from the positive rationals to the positive rationals the function is not onto since there is no rational solution of the equation $x^2 = 2$.

Reasons why Abstract Algebra is valuable to math ed majors (and math majors)

1. Even though many students take a course is discrete math where they study various proof techniques many of them seem not to absorb this material well. Abstract algebra provides them much more practice at this in a different context than discrete math does.
2. High school math teachers should be very adept at modular arithmetic. Cyclic groups is where they learn this well.
3. Group theory is the mathematics of symmetry--a fundamental notion in science, math and engineering. For example, the symmetry group of a molecule reveals some of its possible (or impossible) chemical properties.
4. There are many important practical applications of modular arithmetic that are best understood by viewing the modular arithmetic in a group theory framework. Examples include the check digits on UPC codes on retail items, ISBN numbers on books, and credit card numbers. In many cases the check digit is the inverse of a weighted sum modulo an integer (10 in the case of a UPC number, 11 in the case of an ISBN number, 9 in the case of Visa travelers checks).
5. Many games can be understood by viewing them as permutation groups. Two examples are the 15 puzzle and the Rubik cube.
6. High school math teachers should be adept at looking at data and making plausible conjectures and generalizations. They should also teach their students to do this. This is a skill that can be learned with practice. Groups and rings provide abundant opportunities for developing this skill.
7. Many people are not comfortable with abstract concepts nor adept at abstract reasoning. The ability to think abstractly is a valuable asset. Abstract algebra helps develop this ability.
8. Abstract Algebra is an ideal capstone course for math ed majors and for those who will go on to grad school in math. Throughout the course they review things like 1-1 functions, onto functions (surprisingly few senior math ed majors understand these ideas well); equivalence relations; basic concepts from linear algebra such as how to multiply matrices, properties of determinants, how to compute a determinant, how to compute the inverse of a matrix, how to tell if a matrix has an inverse, linear transformations (which are group homomorphisms); properties of complex numbers; properties of integers (Euclid's lemma, division algorithm, criterion for divisibility by 9 or 11 or 4); math induction (another important topic that many students do not seem to understand well when they begin an abstract algebra course--this is especially the case for statements that do not involve sums of series); and properties of polynomials (division algorithm, remainder theorem, factor theorem, number of zeros is at most the degree, unique factorization).
9. Doing well in an abstract algebra course is a confidence builder and sometimes causes students to think about going on the graduate school. I once had a student who did extremely well in abstract algebra who went to medical school and now has a high position in the Center for Disease Control in Atlanta. About 20 years after she took the course I met her for dinner while I was at a meeting in Atlanta. I jokingly said to her "Did you use any abstract algebra in med school?" She immediately responded by saying "Abstract algebra was very valuable to me in med school." I asked how. She said that whenever she was taking a difficult course she said to herself "If I can get an A in abstract algebra I can get an A is any course." She was perfectly serious. Many people who start out intending to be high school math teachers or even teach high school math for several years decide to go to grad school in math for an advanced degree (I and many others loved the course and wanted to go to grad school to continue studying the subject). Taking abstract algebra and doing well makes such a move more likely and easier to do.

THE GRADE OF INCOMPLETE: The grade 'I' indicates that the student's work in a course is satisfactory thus far but has not been completed as of the end of the semester. It may be given only when

1. the course work is substantially completed
2. the completed portion of a student's work in a course is of passing quality
3. there is documented proof that it is unjust to hold the student to the original time limit for course completion

It is the responsibility of the student who has incurred a grade of Incomplete to fulfill the requirements of that course within a maximum of one calendar year from the date on which the I grade is recorded. After one calendar year, a grade of Incomplete automatically changes to a grade of F on the student's record.

ACADEMIC HONESTY: It is the responsibility of the student to know of the prohibited actions such as cheating, fabrication, plagiarism, academic, and personal misconduct, and thus, to avoid them. All students are held to the standards outlined in the code. Please reference the http://dsa.indiana.edu/Code/ for a complete listing. Any violation may result in serious academic penalty, ranging from receiving a warning, to failing the assignment, to failing the course, to expulsion from the University.

STUDENTS WITH DISABILITIES: If you have a documented disability and need assistance, special arrangements can be made to accommodate most needs. Visit web page http://www.iun.edu/~supportn/contactus.shtml for more information.

This is a tentative schedule that may vary during the semester. Please attend the classes if you want to know about any changes to this schedule! (I will try to keep updated syllabus on the web: http://www.iun.edu/~mathiho)

Final Words

I believe that every student is capable of learning mathematics. Any student entering this course who meets the prerequisites should be successful.

My exams are said to be fair but difficult. Please STUDY. Try to get a good night's rest and eat a nutritious meal before the exam so that you will have the stamina to keep working, if need be, for the full class-period.

Please let me know immediately if you are having any problems that interfere with your progress in this course. I usually check my email daily. I will do all I can to help.

TENTATIVE SYLLABUS

Week	Date	Day	Assign #	Homework Problems	DUE
1	Sep 1	Tuesday	**0**	Chapter 0: Integers and Equivalence Relations 4, 8, 10, 11, 12, 14, 20, 21, 22, 24, 30, 32	
	Sep 3	Thursday			
2	Sep 8	Tuesday			
	Sep 10	Thursday	1	Chapter 1: Introduction to Groups: 2, 3, 6, 10, 12	**0**
3	Sep 15	Tuesday			
	Sep 17	Thursday			
4	Sep 22	Tuesday	**2**	Chapter 2: Groups: 3, 4, 5, 6, 9, 12, 13, 15, 16, 19, 20, 25, 26, 30	**1**
	Sep 24	Thursday			
5	Sep 29	Tuesday			
	Oct 1	Thursday	**3**	Chapter 3: Finite Groups; Subgroups 1, 4, 6, 10, 18, 19, 20, 28, 29, 34, 44, 49, 50	**2**
6	Oct 6	Tuesday			
	Oct 8	Thursday			
7	Oct 13	Tuesday		Review	**3**
	Oct 15	Thursday		**MIDTERM**	
8	Oct 20	Tuesday	**4**	Chapter 4: Cyclic Groups: 3, 7, 12, 18, 20, 22, 30, 31, 36, 42, 51, 64	
	Oct 22	Thursday			
9	Oct 27	Tuesday	**5**	Chapter 5: Permutation Groups 1, 3, 4, 9, 12, 17, 18, 26, 28, 36, 38, 41, 42, 48	
	Oct 29	Thursday			**4**
10	Nov 3	Tuesday			
	Nov 5	Thursday			
	Nov 6	Friday	**Automatic 'W' Deadline**		
11	Nov 10	Tuesday	**6**	Chapter 6: Isomorphisms: 2, 4, 10, 14, 19, 24, 26, 27, 32, 34	**5**
	Nov 12	Thursday			
12	Nov 17	Tuesday			
	Nov 19	Thursday			
13	Nov 24	Tuesday	**7**	Chapter 12: Introduction to Rings: 1, 2, 3, 4, 6, 8, 18, 19, 22, 44	**6**
	Nov 26	Thursday			
14	Dec 1	Tuesday			
	Dec 2	Thursday			
15	Dec 8	Tuesday			**7**
	Dec 10	Thursday		Review	
Finals Week	Dec 15	Tuesday	**FINAL EXAM**		