Midterm-practice ________________

1. Solve the equation:

(a) $4(2y+5) = 3(5y-2)$, Solution is: $\frac{26}{7}$

(b) $\frac{1}{5}x + 2 = 3 - \frac{2}{7}x$, Solution is: $\frac{35}{17}$

(c) $(x-1)(x+1) - 5 = x$, Solution is: $-2, 3$

(d) $x + x^2 + 1 + (x+1)^2 = -2x$, Solution is: $-2, -\frac{1}{2}$

(e) $(5x-7)(2x+1) - 10x(x-4) = 0$, Solution is: $\frac{7}{31}$

(f) $\frac{2}{2x+1} - \frac{3}{2x-1} = \frac{-2x+7}{4x^2-1}$, No solution found.

(g) $\frac{3}{2-x} + \frac{2}{x-1} = \frac{1}{x^2-3x+2}$, Solution is: -2 :

(h) $\frac{3}{2-x} + \frac{2}{x-1} = -2$, Solution is: $\frac{1}{2}, 3$

(i) $\frac{x}{x-2} - \frac{x+3}{x} = 1$, Solution is: $-2, 3$

(j) $\frac{x}{x-2} - \frac{x+3}{x} = 0$, Solution is: 6

(k) $\frac{x}{x-2} - \frac{x+3}{x} = -1$, No solution found.

(l) $\frac{x+6}{x-2} - \frac{x-3}{x+2} = 9$, Solution is: $-\frac{14}{9}, 3$

(m) $\frac{5x}{x-2} + \frac{3}{x} + 2 = \frac{-6}{x^2-2x}$, Solution is: $\frac{1}{7}$

(n) $x^2 + 4x + 2 = 0$, Solution is: $-2 \pm \sqrt{2}$

(o) $6x^2 - x = 2$, Solution is: $-\frac{1}{2}, \frac{2}{3}$

(p) $4x^2 + 81 = 36x$, Solution is: $\frac{9}{2}$

(q) $|x+4| = 11$, Solution is: $7, -15$

(r) $3|x+1| - 2 = -11$, Solution is: $2, -4$

(s) $\sqrt{7-x} = x - 5$, Solution is: 6

(t) $\sqrt{2x+15} - 2 = \sqrt{6x+1}$, Solution is: $\frac{1}{2}$

(u) $\sqrt{11+8x} + 1 = \sqrt{9+4x}$, Solution is: $-\frac{5}{4}$

(v) $\left(\frac{t}{t+1}\right)^2 - \frac{2t}{t+1} - 8 = 0$, Solution is: $-\frac{4}{3}, -\frac{2}{3}$

2. Solve the inequality. Express the solution as an interval.
   (a) $3x - 2 > 14$, Solution is: $\left(\frac{16}{3}, \infty\right)$
   (b) $4 > \frac{2-3x}{7} \geq -2$, Solution is: $\left(-\frac{26}{3}, \frac{16}{3}\right]$
   (c) $(x-4)^2 > x(x+12)$, Solution is: $\left(-\infty, \frac{4}{5}\right)$
   (d) $|2x+5| < 4$, Solution is: $\left(-\frac{9}{2}, -\frac{1}{2}\right)$
   (e) $\frac{3}{|5-2x|} \geq 2$, Solution is: $\left[\frac{7}{4}, \frac{5}{2}\right) \cup \left(\frac{5}{2}, \frac{13}{4}\right]$
   (f) $25x^2 - 9 < 0$, Solution is: $\left(-\frac{3}{5}, \frac{3}{5}\right)$
   (g) $\frac{x-2}{x^2-3x-10} \geq 0$, Solution is: $(-2, 2] \cup (5, \infty)$
   (h) $\frac{1}{x-2} \geq \frac{3}{x+1}$, Solution is: $(-\infty, -1) \cup \left(2, \frac{7}{2}\right]$
   (i) $x^3 > x$, Solution is: $(-1, 0) \cup (1, \infty)$
3. Word problems:
   (a) A worker's take-home pay is $\$492$, after deductions totaling 40% of the gross pay have been subtracted. What is the gross pay? Solution \$820.
   (b) At noon Larry and Linda start driving in the same direction. Larry drives at 30 mph. However, after Larry travelled 60 miles, Linda triples her pace and tries to catch-up with Larry. She finally catches up to him at 4 p.m.
      (i) How long did it take Larry to travel 60 miles? Solution: $t = \frac{60}{30} = 2$ hours
      (ii) How far did **Larry** drive by 4 pm? Solution: $d = (30)(4) = 120$ miles
      (iii) How far did **Linda** drive by 4 pm? Solution: Same = 120 miles
      (iv) How fast did **Linda** drive **before** she decided to triple her speed? Solution: $(v \text{ mph})(2 \text{ hours}) + (3v \text{ mph})(2 \text{ hours}) = 120 \text{ miles}$; $v = \frac{120}{8} = 15$ mph;
      (v) How far did **Linda** drive **before** she decided to triple her speed? Solution: $d = (15)(2) = 30$ miles
      (vi) What was Linda's speed after she decided to triple it? Solution: $3(15) = 45$ mph
      (vii) How far did Linda drive at this triple speed? Solution: $d = (45)(2) = 90$ miles
      (viii) When would Linda catch up with Larry, if she decided to, instead of tripling her speed, start driving at 90mph after 2 pm? Solution: $(30)(t) = (15)(2) + (90)(t-2)$, Solution is: $\frac{5}{2} = 2.5$, the time is 2:30 pm
   (c) How much pure copper must be melted with 100 lb of an alloy that is 40% copper to yield an alloy that is 75% copper? Solution: $(1.00)(x) + (0.40)(100) = (0.75)(100 + x)$, Solution is: 140 lb
   (d) Six hundred people attended the premiere of a motion picture. Adult tickets cost $\$9$, and children were admitted for $\$6$. If box office receipts totaled $\$4800$, how many children attended the premiere? Solution: $(6)(x) + (9)(600 - x) = 4800$, Solution is: 200 children.
   (e) Two children, who are 224 meters apart, start walking toward each other at the same instant at rates of 1.5 m/s and 2 m/s, respectively.
      (i) When will they meet? Solution: $(1.5)t + 2(t) = 224$, Solution is: 64 seconds.

(ii) How far will each have walked? Solution: First = $1.5\,(64) = 96$ m; Second = $2\,(64) = 128$ m.

(f) A bullet is fired horizontally at a target, and the sound of the bullet is heard 1.5 seconds later. If the speed of the bullet is 3300 ft/sec and the speed of sound is 1100 ft/sec, how far away is the target? Solution: $3300\,(t) = 1100\,(1.5 - t)$, Solution is: 0.375; $d = 3300\,(0.375) = 1237.5$ ft.

(g) British sterling silver is copper-silver alloy that is 7.5% copper by weight. How many grams of pure copper and how many grams of British sterling silver should be used to prepared 200 grams of a copper-silver alloy that is 10% copper by weight? Solution: $(1.00)\,(x) + (0.075)\,(200 - x) = (0.10)\,(200)$, Solution is: 5.4 g.

(h) A large grain silo is to be constructed in the shape of a circular cylinder with a hemisphere attached to the top. The diameter of the silo is to be 30 feet, but the height is yet to be determined. Find the height h of the silo that will result in a capacity of $11,250\pi$ ft^2. Solution: $\pi\,(15)^2\,(h - 15) + \frac{1}{2}\frac{4}{3}\pi\,(15)^3 = 11250\pi$, Solution is: $h = 55$ ft.

(i) It takes a boy 90 minutes to mow the lawn, but his sister can mow it in 60 minutes. How long would it take them to mow the lawn if they worked together, using two lawn mowers? Solution: $\frac{1}{90} + \frac{1}{60} = \frac{1}{t}$, Solution is: 36 minutes.

(j) It takes a girl 45 minutes to deliver the newspapers on her route; however, if her brother helps, it takes them only 20 minutes. How long would it take her brother to deliver the newspapers by himself? Solution: $\frac{1}{45} + \frac{1}{t} = \frac{1}{20}$, Solution is: 36 minutes.

(k) A rectangular plot of ground having dimensions 26 feet by 30 feet is surrounded by a walk of uniform width. If the area of the walk is 240 ft^2, what is its width? Solutions: $2\,(26 + x)\,x + 2\,(30 + x)\,x = 240$, Solution is: $-30, 2$; $x = 2$ feet.

(l) The speed of the current in a stream is 5 mi/hr. It takes a canoeist 30 minutes longer to paddle 1.2 miles upstream than to paddle the same distance downstream. What is the canoeist's rate in still water? Solution: $t_{up} = t_{down} + \frac{1}{2}$. $t_{up} = \frac{1.2}{x-5}$, $t_{down} = \frac{1.2}{x+5}$; so $\frac{1.2}{x-5} = \frac{1.2}{x+5} + \frac{1}{2}$, Solution is: $7.0, -7.0$; $x = 7$ mph.

4. Write the expression in form $a + bi$, where a and b are real numbers:
   (a) $(1 - 3i)\,(2 + 5i) \quad = 17 - i$

   (b) $i^{20} - 1 \quad = 0$

   (c) $\frac{3}{2+4i} \quad = \frac{3}{10} - \frac{3}{5}i$

   (d) $(2 + 5i)^3 \quad = -142 - 65i$

   (e) Solve the equation (including the complex solutions): $x^2 + 4x + 13 = 0$, Solution is: $-2 - 3i, -2 + 3i$

   (f) Solve the equation (including the complex solutions): $x^2 - 6x + 13 = 0$, Solution is: $3 - 2i, 3 + 2i$

   (g) Solve the equation (including the complex solutions): $4x^4 + 25x^2 - 36i^2 = 0$, $x = \pm 2i, \pm\frac{3}{2}i$

5. What is the equation of the circle in standard form? What is the radius and the center (if not given already)?
   (a) $x^2 + y^2 - 4x + 6y - 26 = 0$ Solution: $(x - 2)^2 + (y + 3)^2 = \left(\sqrt{39}\right)^2$; $C\,(2, -3)$; $r = \sqrt{39}$

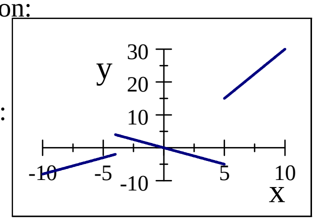
   (b) $x^2 + y^2 + 4y - 117 = 0$ Solution: $x^2 + (y + 2)^2 = 11^2$; $C\,(0, -2)$; $r = 11$

   (c) $2x^2 + 2y^2 - 12x + 4y - 15 = 0$ Solution: $(x - 3)^2 + (y + 1)^2 = \left(\frac{\sqrt{70}}{2}\right)^2$; $C\,(3, -1)$; $r = \frac{1}{2}\sqrt{70}$.

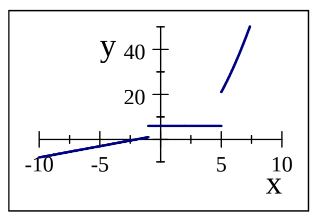
6. Given the information about the straight line, find the equation of the line
   (a) in general form: Point $A(5,-2)$; parallel to the $y-$ axis. Solution: $x=5$
   (b) in general form: Point $A(5,-2)$; parallel to the $y-$ axis. Solution: $y=-2$
   (c) in general form: Point $A(4,0)$; slope $=-4$. Solution: $3x+y=12$
   (d) in slope-intercept form. Through points $A(5,2)$ and $B(-1,4)$. Solution: $y=-\frac{1}{3}x+\frac{11}{3}$
   (e) in slope-intercept form. Point $A(7,-3)$; perpendicular to the line $2x-5y=8$. Solution: $y=-\frac{5}{2}x+\frac{29}{2}$

7. If the point $P(x,y)$ is on the graph of $y=f(x)$, what is the corresponding point on the graph of the given function?
   (a) $P(3,-1)$: $y=2f(2x+1)$ Solution: $2x+1=3$, $x=1$, $2f(2x+1)=-2$, Point $(1,-2)$
   (b) $P(0,5)$; $y=f(x+2)$ Solution: $(-2,5)$
   (c) $P(3,-2)$: $y=2f(x-4)$ Solution: $(7,-4)$
   (d) $P(3,9)$: $y=\frac{1}{3}f\left(\frac{1}{2}x\right)$ Solution: $(6,3)$

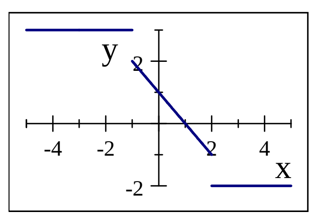
8. Sketch the graph of the following piecewise defined function:
   (a) $f(x)=\begin{cases} 2+x & if \quad x<-4 \\ -x & if \quad -4\le x\le 5 \\ 3x & if \quad x>5 \end{cases}$ Solution:



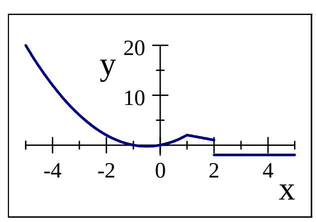
   (b) $f(x)=\begin{cases} 2+x & if \quad x<-1 \\ 6 & if \quad -1\le x\le 5 \\ x^2-4 & if \quad x>5 \end{cases}$ Solution:



   (c) $f(x)=\begin{cases} 3 & if \quad x<-1 \\ -x+1 & if \quad -1\le x\le 2 \\ -2 & if \quad x>2 \end{cases}$ Solution:



   (d) $f(x)=\begin{cases} x^2+x & if \quad x<1 \\ -x+3 & if \quad 1\le x\le 2 \\ -2 & if \quad x>2 \end{cases}$ Solution:



9. Find the domain of
   (a) $f(x)=\sqrt{5+4x}+1$: Solution $5+4x\ge 0$, Solution is: $\left[-\frac{5}{4},\infty\right)$
   (b) $f(x)=\sqrt{1-3x}$ Solution: $1-3x\ge 0$, Solution is: $\left(-\infty,\frac{1}{3}\right]$

(c) $f(x) = \sqrt[3]{1-3x}$ Solution; All real numbers, $(-\infty, \infty)$

(d) $f(x) = \sqrt[3]{3x^2-4x+1}$ Solution; All real numbers, $(-\infty, \infty)$

(e) $f(x) = \sqrt{x^2-5x+6}$ Solution: $x^2-5x+6 \geq 0$, Solution is: $(-\infty, 2] \cup [3, \infty)$

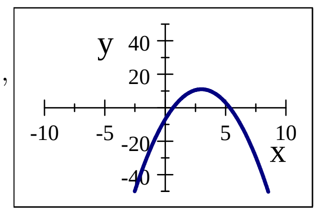
(f) $f(x) = \sqrt{-x^2+5x-6}$ Solution $-x^2+5x-6 \geq 0$, Solution is: $[2, 3]$

(g) $f(x) = \dfrac{1}{\sqrt{-x^2+5x-6}}$ Solution: $-x^2+5x-6 > 0$, Solution is: $(2, 3)$

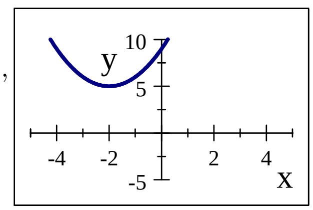
(h) $f(x) = \dfrac{1}{-x^2+5x-6}$ Solution: $-x^2+5x-6 \neq 0$, Solution is: $(-\infty, 2) \cup (2, 3) \cup (3, \infty)$

10. For the given quadratic: Find the zeros (roots) of $f(x)$; Find the maximum or minimum value; Sketch the graph of f

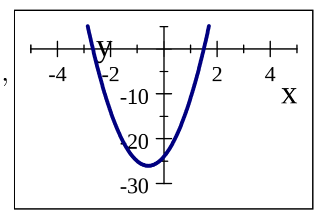
(a) $f(x) = 12x - 2x^2 - 7$ Solution: roots $3 - \frac{1}{2}\sqrt{22}, \frac{1}{2}\sqrt{22} + 3$, Max = 11,



(b) $f(x) = x^2 + 4x + 9$, Solution: roots - none, Min = 5,



(c) $f(x) = 6x^2 + 7x - 24$, Solution: roots - $-\frac{8}{3}, \frac{3}{2}$, Min = $-\frac{625}{24}$,



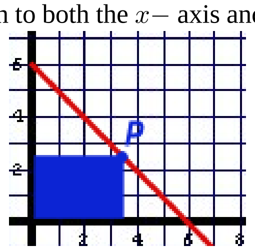
11. Word problems:

(a) Two numbers add to 11. What is the largest possible value of their product? Solution $Max: 30.25$

(b) Among all rectangles having a perimeter of 20 meters, find the dimensions of the one with the largest area. Solution: $Max: 25\ \text{m}^2$

(c) The point P lies in the first quadrant on the graph of the line $y = 6 - x$. From the point P, perpendiculars are drawn to both the $x-$ axis and $y-$ axis. What is the largest possible area for the rectangle thus formed?

Solution:



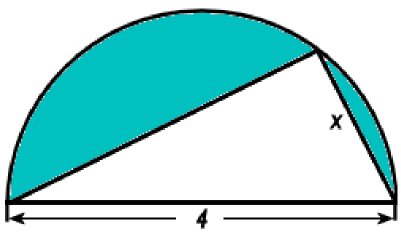
$A = xy = x(6-x) = -x^2 + 6x$
Max at $x = -\frac{b}{2a} = -\frac{6}{-2} = 3$
Max area = $(6-3)(3) = 9$

(d) Suppose you have \$8800 available to fence off a rectangular area finds that the fencing in the east-west direction will require extra reinforcement due to strong prevailing winds. Because of this, the cost of fencing in the east-west direction will be \$10 per (linear) yard, as opposed to a cost of \$5 per yard for fencing in the north-south direction. Find the dimensions of the rectangle with the largest possible area that

can be built!
Solution:

$C = 2\,(\$10l) + 2\,(\$5h) = \$8800 : \$20l + \$10h = \$8800$
$h = \frac{8800-20l}{10} = 880 - 2l$
$Area = h\,(880 - 2l) = -2h^2 + 880h$
$Max : h = -\frac{b}{2a} = -\frac{880}{-4} = 220 : l = 440$
Max Area is $A = 220 \times 440 = 96\,800$

(e) A triangle is inscribed in a semicircle of diameter 4, as shown in the figure. Show that the minimum possible value for the shaded area around the triangle is $2\,(\pi - 2)$. (HINT: The area of the shaded region is minimum when the area of the triangle is a maximum. Find the value of x that maximizes the **square** of the area of the triangle. This will be the same x that maximizes the area of the triangle.)
Solution:



$Total\_Semicircle = \frac{1}{2}\pi 2^2 = 2\pi$
$Triangle\_Area = \frac{1}{2}x\left(\sqrt{4^2 - x^2}\right)$
$Triangle\_Area^2 = \frac{1}{4}x^2\left(16 - x^2\right) = \frac{1}{4}\left(-\left(x^2\right)^2 + 16x^2\right)$
$Triangle\_Area^2$ is max for: $x^2 = -\frac{b}{2a} = -\frac{16}{-2} = 8$
$Triangle\_Area = \frac{1}{2}\sqrt{8}\sqrt{16-8} = \frac{1}{2}8 = 4$
$Shaded\_Area = \frac{1}{2}\pi\,(2)^2 - 4 = 2\pi - 4 = 2\,(\pi - 2)$

12. For the given functions find: (i) $(f \circ g)(x)$; (ii) $(g \circ f)(x)$ (iii) $f(g(-2))$ (iv) $g(f(x))$

(a) $f(x) = 2x - 5$; $g(x) = 3x + 7$; Solution is: (i) $6x + 9$, (ii) $6x - 8$, (iii) -3, (iv) 10

(b) $f(x) = 3x^2 + 4$, $g(x) = 5x$; Solution: (i) $75x^2 + 4$, (ii) $15x^2 + 20$, (iii) 304, (iv) 155

(c) $f(x) = 4x$, $g(x) = 2x^3 - 5x$, Solution: (i) $8x^3 - 20x$, (ii) $128x^3 - 20x$, (iii) -24, (iv) 3396

13. Find (i) $(f \circ g)(x)$ and the domain of $(f \circ g)(x)$ (ii) $(g \circ f)(x)$ and the domain of $(g \circ f)(x)$

(a) $f(x) = x^2 - 4$; $g(x) = \sqrt{x}$; Solution: (i) $3x - 4$, Domain: $[0, \infty)$, (ii) $\sqrt{3x^2 - 12}$, Domain $(-\infty, -2] \cup [2, \infty)$

(b) $f(x) = \sqrt{x-2}$; $g(x) = \sqrt{x+5}$; Solution: (i) $\sqrt{\sqrt{x+5} - 2}$, Domain: $[-1, \infty)$, (ii) $\sqrt{\sqrt{x-2} + 5}$, Domain $[2, \infty)$

(c) $f(x) = \dfrac{x-1}{x-2}$, $g(x) = \dfrac{x-3}{x-4}$, Solution: (i) $\dfrac{\dfrac{x-3}{x-4} - 1}{\dfrac{x-3}{x-4} - 2} = -\dfrac{1}{x-5}$, Domain $x \neq 4, x \neq 5$, (ii) $\dfrac{\dfrac{x-1}{x-2} - 3}{\dfrac{x-1}{x-2} - 4} = \dfrac{2x-5}{3x-7}$, Domain: $x \neq 2$, $x \neq \dfrac{7}{3}$