Math 118 Fall 2007

Practice Final Exam

Name:____________________________________

PLEASE READ THESE INSTRUCTIONS

This practice test consists of 30 partial credit problems.

1. Solve the following system:

$$\begin{cases} 2x - 6y = 2 \\ 3x - 5y = 11 \end{cases}$$

2. Suppose that the supply and demand equations for a certain type of clothing are given by:

$$p = 0.8q + 5 \quad \text{Supply}$$

$$p = -1.8q + 12 \quad \text{Demand}$$

Find the equilibrium price and quantity.

3. Solve the following system:

$$\begin{cases} 9x - 15y = -6 \\ -6x + 10y = 4 \end{cases}$$

4. Solve the following system using augmented matrices:

$$\begin{cases} 3x_1 + 2x_2 = 4 \\ 2x_1 - x_2 = 5 \end{cases}$$

5. Solve the following system using Gauss-Jordan elimination:

$$\begin{cases} 2x_1 - x_2 - 4x_3 = 9 \\ x_1 - 3x_2 = 8 \end{cases}$$

6. Solve the following system using Gauss-Jordan elimination:

$$\begin{cases} 3x_1 - x_2 = 2 \\ 4x_1 + 5x_2 = 0 \\ x_1 - 2x_2 = 1 \end{cases}$$

7. Use row operations to change the following matrix to its reduced form:

$$\begin{bmatrix} 1 & 0 & 5 & | & 1 \\ 0 & 1 & -1 & | & -2 \\ 0 & 0 & -3 & | & 0 \end{bmatrix}$$

8. A corporation wants to lease a fleet of 25 airplanes. There are three types of planes available: small, which carries 12 passengers, medium, which carries 18 passengers and large, with the capacity of 26 passengers. The company needs to transport 330 passengers. Set up a linear system that answers the following question: How many of each type of plane should be leased? Write the augmented matrix of the system.

9. Given the following matrices:

$$A = \begin{bmatrix} -2 & -1 \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 3 & -1 \\ 1 & 3 \end{bmatrix} \quad \text{and } C = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}$$

compute $AC - 3B$.

10. Find x and y so that:

$$\begin{bmatrix} 5 & 3x \\ 2x & -4 \end{bmatrix} + \begin{bmatrix} 1 & -4y \\ 7y & 4 \end{bmatrix} = \begin{bmatrix} 6 & -7 \\ 5 & 0 \end{bmatrix}$$

11. Find $a, b, c,$ and d so that:

$$\begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$$

12. Determine the inverse of the following matrix, if it exists:

$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix}$$

13. Determine the inverse of the following matrix, if it exists:

$$\begin{bmatrix} -1 & -2 & 2 \\ 4 & 3 & 0 \\ 4 & 0 & 4 \end{bmatrix}$$

14. Find x_1 and x_2:

$$\begin{bmatrix} 1 & -1 \\ 1 & -2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

15. Solve the following system using matrix equations:

$$\begin{cases} 2x_1 + x_2 = k_1 \\ x_1 + x_2 = k_2 \end{cases}$$

in the following cases:

(A) $k_1 = -2, \quad k_2 = 1$

(B) $k_1 = -1, \quad k_2 = 3$

16. Assume that A and B are $n \times n$ matrices, and that C and D are $n \times 1$ matrices.

Assume also that all necessary inverses exist. Solve the matrix equation:

$$AX - X = D$$

17. Solve the system of linear inequalities:

$$\begin{cases} x - 2y \leqslant 12 \\ 2x + y \geqslant 4 \end{cases}$$

18. Solve the system of linear inequalities and find the coordinates of each corner:

$$\begin{cases} 2x + y \leqslant 10 \\ x + 2y \leqslant 8 \\ x \geqslant 0 \\ y \geqslant 0 \end{cases}$$

19. Solve the system of linear inequalities and find the coordinates of each corner:

$$\begin{cases} -x + 3y \geqslant 1 \\ 5x - y \geqslant 9 \\ x + y \leqslant 9 \\ x \leqslant 5 \end{cases}$$

20. Minimize and maximize $z = 10x_1 + 30x_2$ subject to:

$$\begin{cases} 2x_1 + x_2 \geqslant 16 \\ x_1 + x_2 \geqslant 12 \\ x_1 + 2x_2 \leqslant 14 \\ x_1, x_2 \geqslant 0 \end{cases}$$

21. Minimize and maximize $z = x_1 - x_2$ subject to:

$$\begin{cases} x_1 - 2x_2 \leqslant 0 \\ 2x_1 - x_2 \geqslant 6 \\ x_1, x_2 \geqslant 0 \end{cases}$$

22. The officers of a high school senior class are planning to rent busses and vans for a class trip. Each bus can transport 40 students, requires 3 chaperons, and costs $1,200 to rent. Each van can transport 8 students, requires 1 chaperon, and costs $100 to rent. Since there are 400 students in the senior class, the officers must plan to accommodate at least 400 students. At most 36 chaperons can be used. How many vehicles of each type should be rented in order to minimize the transportation costs?

23. For the following system, introduce slack variables and create a table listing each basic solution:

$$\begin{cases} 2x_1 + x_2 \leqslant 50 \\ x_1 + 2x_2 \geqslant 40 \\ x_1, x_2 \geqslant 0 \end{cases}$$

24. For the following system, introduce slack variables and create a table listing each basic solution:

$$\begin{cases} x_1 + x_2 \leqslant 16 \\ 2x_1 + x_2 \geqslant 20 \\ x_1, x_2 \geqslant 0 \end{cases}$$

25. Given the following simplex tableau, choose the pivot, if possible, and perform a pivot operation:

$$\left[\begin{array}{cccccc|c} x_1 & x_2 & s_1 & s_2 & s_3 & P & \\ \hline 0 & 0 & 2 & 1 & 1 & 0 & 2 \\ 1 & 0 & -4 & 0 & 1 & 0 & 3 \\ 0 & 1 & 5 & 0 & 2 & 0 & 11 \\ \hline 0 & 0 & -6 & 0 & -5 & 1 & 18 \end{array}\right]$$

26. For the following problem, set up the initial simplex tableau:

A small company manufactures three types of motherboards. Type A requires 2 hours of fabrication and 1 hour of assembly, type B requires 3 hours of fabrication and 1 hour of assembly, type C requires 2 hours of fabrication and 2 hours of assembly. The company has 1000 labor-hours of fabrication, and 800 labor-hours of assembly. The profit for each motherboard is \$7, \$8, and \$10 respectively. How many motherboards of each type should the company manufacture, in order to maximize the profit?

27. For a Markov chain, the transition matrix P and the initial state S_0 are given below. Find S_2.

$$P = \begin{bmatrix} .8 & .2 \\ .3 & .7 \end{bmatrix}, \quad S_0 = \begin{bmatrix} .5 & .5 \end{bmatrix}$$

28. A Markov process has two states $A,$ *and* B. The probability of going from state A to state A in one trial is 0.1, the probability of going from state B to state B in one trial is 0.7. Determine the transition matrix (you can draw the transition diagram). What is the probability of going from state A to state B in two trials?

29. Given a Markov process with the following transition matrix, find the station-

ary matrix S and the limiting matrix $\bar{P}$.

$$P = \begin{bmatrix} .3 & .7 \\ .2 & .8 \end{bmatrix}$$

30. Consumers in a certain state can choose between three different phone companies, GTT, NCJ, and Dash. Aggressive marketing by all three companies results in a continual shift of customers among the three services. Each year, GTT loses 5% of its customers to NCJ and 20% of its customers to Dash, NCJ loses 15% of its customers to GTT and 10% of its customers to Dash, and Dash loses 5% of its customers to GTT and 10% of its customers to NCJ. Assuming that these percentages remain valid over a long period of time, what is each company's expected market share in the long run?