

M118 - Fall 2009 Practice Midterm

Instructions

1. The actual exam will consist of problems *similar* to the following.
2. You may use a calculator.
3. You may have notes on one sheet of notebook paper (both sides).

Practice Problems

1. True or False
 - (a) _____ $\{1, 2, 3\} = \{3, 1, 2\}$
 - (b) _____ $\{1, 2, 3\} \subset \{1, 2, 3\}$
 - (c) _____ $\{1, 2, 3\} \subseteq \{1, 2, 3\}$
2. $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$, $X = \{2, 3, 4, 5, 6\}$, $Y = \{3, 5, 7, 9\}$, and $Z = \{2, 4, 11, 12\}$. List the members of the following sets.
 - (a) $X \cap Y$
 - (b) $\bar{X} \cap (Y \cup Z)$
 - (c) Draw a Venn diagram and shade in the above region
3. Of the cars sold during the month of July
 - 90 had heated sets
 - 100 had GPS
 - 75 had satellite radio
 - 5 had all three of these
 - 20 cars had none of these extras
 - 20 cars had only heated seats
 - 60 cars had only GPS
 - 30 cars had only satellite radio
 - 10 cars had both GPS and satellite radio
 - (a) How many cars had both satellite radio and heated seats?
 - (b) How many had neither satellite radio nor heated seats?

- (c) How many cars were sold during July?
4. Given that $n(A) = 9$, $n(B) = 11$, $n(A \cap B) = 5$, $n(\overline{A}) = 13$
- (a) Find $n(A \cap \overline{B})$.
- (b) Find $n(\overline{A} \cap B)$.
- (c) Find $n(\overline{A \cup B})$.
5. Evaluate the following without using a calculator.
- (a) $6!$
- (b) $P(7, 2)$
- (c) $C(9, 4)$
6. A basketball team consists of 5 *distinct* positions. If there are 8 players to choose from, how many starting teams are possible?
7. How many different 9 letter words (real or imaginary) can be formed from the letters in the word *committee*?
8. A club consists of 12 women and 9 men needs to form a committee of 5 people for the annual fund raiser.
- (a) How many different committees can be formed?
- (b) How many committees can be formed consisting of 3 women and 2 men?
9. A securities analyst is reviewing the performance of a group of electric utilities. He finds that 65% have increased sales and 25% have increased earnings. 15% have increased neither sales nor earnings. A utility is selected at random from the group. What is the probability that it has increased both sales and earnings?
10. Suppose that the probability for rain is 0.30. What are the odds against it?
11. Suppose the past records in a large city produced the following probability data on a driver being in an accident on the last day of a Memorial Day weekend:

	Accident A	No Accident \overline{A}
Rain (R)	10	134
No Rain (\overline{R})	6	250

- (a) Find $P(\overline{A})$

- (b) Find $P(A|\bar{R})$
 (c) Find $P(\bar{R}|A)$
12. Suppose an urn contains red balls marked 1,2,3; a blue ball marked 4; and white balls marked 5,6,7,8. A ball is drawn at random.
- (a) What is the probability the ball is red?
 (b) If the ball is known to have an even number, what is the probability that it is red?
13. A factory produced 25 new HD LCD television sets of which 5 were defective. A store ordered 5 of these sets for their sale. What is the probability that 2 of these sets are defective?
14. A basketball player is to shoot free throws until she misses. If the probability that she makes a free throw is 70% and each attempt is independent of the other, then what is the probability that
- (a) She will take exactly one shot?
 (b) She will take exactly three shots?
15. The following data represents the numbers, in thousands, of persons with and without health insurance coverage by age in the year 2005.

	Age			
	< 18	18-44	45-64	≥ 65
Insured H	65,675	83,549	63,038	35,046
Not Insured \bar{H}	8,310	27,068	10,740	459

- (a) What is the probability that a randomly selected individual who is less than 18 years old has no health insurance?
 (b) What is the probability that a randomly selected individual who has no health insurance is less than 18 years old?
16. In a survey of 100 people categorized as drinkers or nondrinkers, with or without a liver ailment, the following data was obtained:

	Liver Ailment A	No Liver Ailment \bar{A}
	Drinkers D	52
Nondrinkers \bar{D}	8	22

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- (a) Are the events D and A independent?
- (b) Are the events D and \bar{A} independent?
17. Let $P(A)=0.3$ and $P(B)=0.4$.
- (a) Find $P(A \cup B)$ if $P(A \cap B) = 0.2$.
- (b) Find $P(A \cup B)$ if A and B are independent.
- (c) Find $P(A \cup B)$ if A and B are mutually exclusive.
18. Students at GSU are being tested for the North Amazon virus. If a student is infected, the test is positive 90% of the time, and if a student is not infected, the test is negative 90% of the time. Suppose that 2% of the students are infected with the virus. If a student is chosen at random, find the probability that the test is positive.
19. In conducting a political poll, a pollster divides the nation into four sections: Northeast (N), containing 40% of the population; South (S), containing 10% of the population; Midwest (M), containing 25% of the population; and West (W), containing 25% of the population. From the poll it is found that in the next election 40% of the people in the Northeast say they will vote Republican, in the South 56% of the people will vote Republican, in the Midwest 48% of the people will vote Republican, and in the West 52% of the people will vote Republican.
- (a) What is the probability that a person chosen at random will vote Republican?
- (b) Assuming a person chosen at random votes Republican, what is the probability that he or she is from the Northeast?
20. The 2000 United States census showed that 87.6% of the people were younger than 65 years old. If 10 people were randomly selected from the United States,
- (a) What is the probability that exactly 8 of these people would be younger than 65?
- (b) What is the probability that at least one person is *more* than 65?

Answers

1. True or False

(a) True

(b) False

(c) True

2. $\bar{X} = \{1, 7, 8, 9, 10, 11, 12\}$, $Y \cup Z = \{2, 3, 4, 5, 7, 9, 11, 12\}$

(a) $X \cap Y = \{3, 5\}$

(b) $\bar{X} \cap (Y \cup Z) = \{7, 9, 11, 12\}$

See end of answer section for venn diagrams

3. July car sales

(a) 40

(b) 80

(c) 205

See end of answer section for venn diagrams

4. For the basic regions

(a) $n(A \cap \bar{B}) = 4$

(b) $n(\bar{A} \cap B) = 6$

(c) $n(\overline{A \cup B}) = 7$

5. Expected computations

(a) $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

(b) $P(7, 2) = \frac{7!}{(7-2)!} = \frac{7 \cdot 6 \cdot 5!}{5!} = 42$

(c) $C(9, 4) = \frac{9!}{(9-4)! \cdot 4!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{5! \cdot 4!} = \frac{3024}{24} = 126$

6. $P(8, 5) = 6,720$

7. $\frac{9!}{1! \cdot 1! \cdot 2! \cdot 1! \cdot 2! \cdot 2!} = 45,360$

8. The number of committees are

- (a) $C(21, 5) = 20,349$
(b) $C(12, 3) \cdot C(9, 2) = 7,920$

9. $P(IS \cap IE) = 0.05$

See end of answer section for venn diagram (Listed as problem 10, s/b 9)

10. odds against = 7 to 3

11. Answers

- (a) Find $P(\bar{A}) = \frac{384}{400}$
(b) Find $P(A|\bar{R}) = \frac{6}{256}$
(c) Find $P(\bar{R}|A) = \frac{6}{16}$

12. If one ball is drawn

- (a) $P(R) = \frac{3}{8}$
(b) $P(R|E) = \frac{1}{4}$

13. $\frac{C(5, 2) \cdot C(20, 3)}{C(25, 5)} = 0.2146$

14. S = success, F=failure

- (a) $P(F) = 0.3$
(b) $P(S \cap S \cap F) = 0.7 \cdot 0.7 \cdot 0.3 = 0.147$

15. Health Insurance

- (a) $P(\bar{H} | < 18) = \frac{8,310}{73,985} = 0.1123$
(b) $P(< 18 | \bar{H}) = \frac{8,310}{46,577} = 0.1784$

16. There are two different ways to show the events are independent. Each is shown below. You may use either method on the exam.

(a) No; $P(D|A) = \frac{n(D \cap A)}{n(A)} = \frac{52}{60} = 0.8667 \neq P(D) = \frac{70}{100}$

$$(b) \text{ No; } P(D \cap \bar{A}) = \frac{18}{100} \neq P(D) \cdot P(\bar{A}) = 0.7 \cdot .4$$

17. Answers

$$(a) P(A \cup B) = .50$$

$$(b) P(A \cup B) = .58$$

$$(c) P(A \cup B) = .70$$

$$18. P(+) = 0.02 \cdot 0.9 + 0.98 \cdot 0.1 = 0.116$$

19. Voting

$$(a) P(R) = .4 \cdot .4 + .1 \cdot .56 + .25 \cdot .48 + .25 \cdot .52 = .466$$

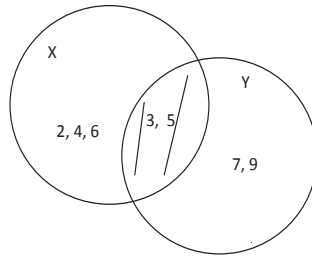
$$(b) P(N|R) = \frac{P(N \cap R)}{P(R)} = \frac{P(N) \cdot P(R|N)}{P(R)} = \frac{0.4 \cdot 0.4}{0.466} = 0.343$$

20. The probabilities are

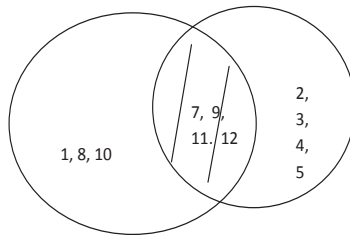
$$(a) C(10, 8) \cdot (0.876)^8 \cdot (1 - 0.876)^2 = 0.2399$$

$$(b) 1 - C(10, 10)(0.876)^{10} \cdot (0.124)^0 = 0.7339$$

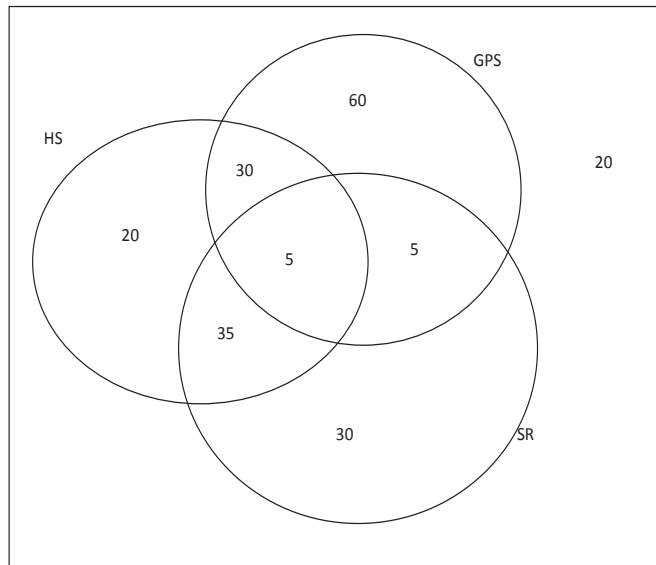
2(a)



2(b)



3.



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